An Economic Analysis of Secured Non-Recourse Debt

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Executive Summary

This research focuses on the economics of secured non-recourse debt, and most particularly on the differences between standard non-recourse secured debt and non-recourse bankruptcy remote secured debt. It examines the debt on a stand-alone basis as well as in a multi-asset, multi-debt financed firm.

A model is developed where assets are subject to two sources of risk: cash flow uncertainty and obsolescence risk. Obsolescence risk is properly interpreted as a determinant of the asset's real capitalization (cap) rate. Lender underwriting standards establish an upper bound on LTV and a lower bound on DSC, and thus constrain the loan proceeds and determine the cost of debt capital.

To establish a foundation for further analysis, we first consider debt funding of a single asset, limited liability firm, and show there are three lending regimes that emerge depending the relative magnitude of the real cap rate. In the first two regions in which the cap rate is high or moderate, the LTV constraint binds due to the low to moderate asset value relative to cash flow. The difference between the two regimes is that cash flow default can occur in the second regime. This happens because a relatively lower cap rate increases loan proceeds based on a binding LTV constraint, which in turn increases the DSC ratio due to increased coupon payments. As a result, higher coupon payments increase the risk of payment default. In a third, low cap rate/high asset value regime, the DSC ratio constraint is binding due to the low cash flow relative to asset value. We show that in this case a wedge can exist between a borrower's cash flow default and strategic default incentive. As a result of the conflicting incentive, the lender may impose a DSC ratio default provision to provide it the option for "strategic liquidation." Novel comparative statics are derived for LTV and DSC ratios as they depend on the real cap rate.

The model is then extended to consider the multi-asset firm. In this firm there is an in-place asset that is financed with unsecured debt. We then consider a new investment opportunity which is financed with one of four alternative debt structures: i) Non-recourse bankruptcy remote secured debt, ii) Standard non-recourse secured debt; iii) Junior unsecured debt, or iv) *Pari-passu* unsecured debt. We show that bankruptcy remote secured debt is immune from wealth transfer/underinvestment problems, whereas standard non-recourse secured debt is not. Rather unintuitively, the size of the wealth transfer to the in-place debtholder is shown to be increasing in the diversity of asset cash flows; that is, asset diversification within the firm is bad when the firm chooses to finance with this type of debt. In the multi-asset, multi-debt financed firm with a mix of different debt structures, bankruptcy remote secured debt is shown to be a stable companion to other debt forms, albeit that it significantly transforms the character of the in-place debt when added to the firm's capital structure. Standard non-recourse secured debt is, in contrast, rather unstable and an uneasy companion to in-place unsecured debt. In the case of *pari-passu* unsecured debt, we show the firm can actually generate greater proceeds at lower cost with the issuance of this type of debt as compared to bankruptcy remote debt. There is, however, a significant wealth transfer to the in-place debtholder with the *pari-passu* unsecured debt, whereas no wealth transfer occurs with the bankruptcy remote secured debt.

Lastly, we analyze the GGP bankruptcy through the lens of our model. Our analysis indicates there was little benefit to retaining GGP management and reviving the firm as a going concern. This leads us to conclude that, although GGP was overlevered and exhibited poor financial management in the months and years leading up to its bankruptcy, their liberal use of bankruptcy remote secured debt was not *ex ante* inefficient. In fact, the use of this type of debt seemed well suited to many of the assets held by GGP, as the assets were are not management specific and created little or no incremental value that was specific to the firm.

An Economic Analysis of Secured Non-Recourse Debt

I. Introduction

Secured non-recourse debt is an important source of funding in the US economy. Even putting aside residential mortgage debt outstanding in the US that is somewhere in the $7 trillion range, a recent Morgan Stanley (2018) report estimates there is $3.1 trillion of non-recourse commercial mortgage debt outstanding in the US. Oil and gas, shipping, and infrastructure are examples of other industries that commonly issue debt on secured non-recourse basis, many times referring to it as “project finance.” All in, and ignoring all forms of secured consumer debt outstanding, it has been estimated that secured non-recourse debt outstanding used for business purposes easily exceeds the sum total of corporate unsecured debt issued and outstanding in the US.

Yet little known about the unique economic characteristics of secured non-recourse debt issued by firms to finance asset ownership and new investment. This is in part because of the limited liability nature of the debt structure, where analysis generally proceeds on a project finance basis *as if* the firm was comprised of a single asset financed with a single source of debt. But firms, enterprises and other going concerns typically hold multiple assets and regularly make new investments. Is secured non-recourse debt efficient in the financing of new investment when there are other forms of in-place debt? Is in-place secured non-recourse debt accommodative to other forms of new debt issuance, and what are the debt contracting externalities, if any, across the firm? These are the kinds of issues that interest us in this paper.

Even within the category of secured non-recourse debt, there are important differences between what we will call standard non-recourse (S-NR) secured debt and non-recourse bankruptcy remote (NR-BR) secured debt. These differences have not, however, been clearly articulated in the economics and finance literature, and are often overlooked in industry practice. A central focus of this study is to examine and highlight the economic differences between S-NR and NR-BR secured debt as part of a larger effort to begin to understand why firms such as private commercial real estate enterprises commonly issue non-recourse secured debt to finance asset ownership and new investment, while other types of firms and industries do not.[1]

To conduct our analysis we develop a flexible and robust, yet relatively simple, multi-period model that accommodates a variety of debt structure and real asset characteristics. Time is discrete in our model, and there is an infinite investment horizon. The firm we consider is financially constrained, in the sense that the (risk-adjusted) cost of outside equity capital is elevated relative to the cost of debt capital. Inside equity is available to finance investment, but is scarce and comes at a non-trivial shadow cost, which creates a preference for debt finance. Assets exhibit two distinct sources of risk: cash flow uncertainty and obsolescence risk, where obsolescence risk captures deterioration in asset cash flow productivity over the longer run. The modeling of cash flow uncertainty and obsolescence risk in combination has been rare in the literature, where this enhancement greatly enriches the analysis.

Debt maturity is finite and extends beyond one period in our model, with periodic coupon payments and a terminal loan payoff required as part of the debt funding structure. This feature is also novel, yet completely realistic. Most tractable models of risky debt either assume either single-period debt with interest and principal due at the end of the period, zero coupon debt in the spirit of Merton (1974), with only a single promised payoff at the debt's finite maturity date, or perpetual coupon debt in the spirit of Leland (1994), which has no maturity date and hence no term payoff amount.

Another novel, yet realistic, feature of our model is the imposition of lender underwriting standards which limit the quantity of debt available to fund investment. Underwriting standards are formalized as lender participation constraints, of which there are two. First, there is a total loan payoff constraint, which functions as a maximum loan-to-value (LTV) ratio constraint, and second, there is a coupon payment constraint, which functions as a minimum debt service coverage (DSC) ratio constraint. Given the borrower's objective of maximizing debt issuance proceeds, one or the other of these lender participation constraints will always be binding.

The first step in our analysis is to consider the financing of a single asset with a single source of debt. The firm in this case is limited liability, implying that debt financing can be characterized as secured and non-recourse,

[1] Bao and Kolasinski (2016) state that non-financial C-corporations (aka publicly traded industrial firms) rarely issue non-recourse secured debt, and when they do it is often structured to function like senior unsecured debt.

and of the stand-alone "project finance" variety. Underwriting standards as described above apply to an asset whose characteristics depend on its cash flow uncertainty and obsolescence risk.

We show the existence of three different lending regimes that endogenously emerge from the model structure. In the first identified regime, asset obsolescence risk is high as compared to cash flow uncertainty, implying the asset has a high cap rate. In this case the LTV underwriting constraint binds due to the low asset value relative to cash flow. Lending in this regime results in only term default. A second regime is characterized by moderate asset cap rates, implying higher asset valuations relative to those that occur in the first regime. The LTV ratio continues to bind in this case. What changes is that cash flow default can occur prior to the end of the loan term. This happens because the increased asset value results in a larger loan amount, which consequently increases the coupon payment on the loan. The higher coupon payment reduces the DSC ratio to increase the likelihood of a cash flow default. We show that cash flow default is incentive compatible for the firm in this region, meaning that default would occur even if the firm had the liquidity to fund the coupon payment. The third regime is characterized by low cap rates and high value assets. In this region the DSC ratio constraint binds due to low cash flow relative to asset value, causing the LTV ratio to be scaled back. Cash flow default also occurs in this region.

Interestingly, in this third regime, when the cap rate is particularly low (e.g., Manhattan office property), liquidity-driven cash flow default is no longer incentive compatible for the firm. That is, the firm would prefer to fund the coupon payment gap with available liquidity, since continuation value exceeds the coupon payment funding gap. Thus, in this case, there is a distinction between liquidity/cash flow default and strategic default. If, at origination, the lender believes that the borrower cannot or will not bridge the coupon funding gap, the integrity of the loan structure remains intact. However, should the lender anticipate the borrower will in fact bridge any funding gap, loan terms will adversely change for the borrower, resulting in reduced loan proceeds and an increased credit spread. To counter this, the firm may commit to a loan provision that blocks bridge funding, thus restoring the original loan terms. The loan provision that does this a DSC ratio default provision, which declares the borrower in default should the ratio fall below 1.0. The model thus explains how a loan provision of this type emerges endogenously, where, surprisingly, it provides the lender the option to liquidate the loan even when the borrower is prepared to cure. Our model suggests the lender will in fact exercise its option, as liquidation/foreclosure is preferred because it generates a higher debt payoff than the expected debt payoff from continuation at the original loan terms. We call this outcome "strategic liquidation" or "strategic foreclosure."

Novel comparative static relations with respect observed LTV and DSC ratios are presented as they depend on cash flow uncertainty and obsolescence risk, which in turn determine the three cap rate-asset value regimes. We show, for example, that in the high cap rate region (regime 1), LTV ratio is decreasing in both cash flow uncertainty and obsolescence risk, whereas the DSC ratio is increasing in obsolescence risk but is decreasing in cash flow uncertainty. In contrast, in the low cap rate region (regime 3), LTV ratio is decreasing in cash flow uncertainty but is increasing in obsolescence risk, whereas the DSC ratio is decreasing in cash flow uncertainty but is insensitive to obsolescence risk.

To better understand the LTV ratio results, consider office property located in mid-town Manhattan (NYC – low cap rate regime) versus suburban St. Louis (SSL – high cap rate regime). NYC property generates low cash flow relative to asset value, whereas the opposite is true with SSL property. Although no clear prediction emerges as to whether LTV's are higher or lower in NYC versus SSL, LTV is predicted to decrease in the real cap rate in SSL, due to collateral asset value being the determining factor in the loan proceeds. This is not the case in NYC, where LTV is predicted to increase for an increase in the cap rate. This happens because an increase in cash flow relative to asset value relaxes the binding DSC ratio constraint, causing loan proceeds to increase significantly.

We then extend the model to consider the multi-asset firm. In this case, we start with a firm that owns one asset (or homogeneous group of assets), where the asset is financed with unsecured (UN) debt. Underwriting standards are extended to allow for both the individual and joint application of lender participation constraints. In this multiple asset case, cash flow uncertainty and obsolescence risk are common across the assets, where now we introduce variation in the scale of new investment and an arbitrary correlation structure between asset cash flows.

In this analysis we distinguish between NR-BR and S-NR secured debt. Both debt structures are non-recourse and reference a specified asset as security. The primary difference between the two debt forms is that NR-BR debt is completely insulated from the reach of outside claimholders from the within the broader firm—particularly outside debtholders. As a consequence, no adverse wealth transfer from equity to in-place debtholders occurs with new investment under any state of the world, implying no underinvestment concerns. This is not the case with S-NR, where outside debtholders have legal standing to reach over and grab available equity should debt payment shortfalls occur in other parts of the firm. This creates the potential for a wealth transfer from equity to the in-place debt, which can adversely impact new investment.

We further show that, should the firm exclusively issue S-NR debt, then the firm is effectively comprised of NR-BR debt, since the non-recourse feature blocks any of the in-place debtholders from reaching outside of the pledged security structure to supplement their payoffs. We also show how the issuance of NR-BR debt to finance new investment changes the character of in-place debt of any form other than NR-BR, transforming it into a NR-BR debt form.

In the more formal, detailed analysis, four debt funding structures are considered as alternatives to finance the new investment opportunity. These four debt funding structures are: i) NR-BR secured debt, ii) S-NR secured debt, iii) junior unsecured (J-UN) debt, and iv) *pari-passu* unsecured (PP-UN) debt. In all cases we assume that UN debt is in place.

The case of NR-BR secured debt issuance is perhaps the most straightforward, given that its structure allows for analysis to occur on a stand-alone basis. The equity-to-debt wealth transfer channel is, as previously discussed, completely shut down with this issuance of NR-BR debt. The low cap rate/high asset value sub-case identified early is relevant here, where *within-firm liquidity* can potentially be generated to fund coupon payment shortfalls when early term cash flow default is no longer incentive compatible for the firm. In this case, the firm may have an incentive call the loan on the performing asset, sell it, and redistribute the excess proceeds to subsidize the poorly performing asset.

This scenario illustrates a perverse incentive in which a firm potentially liquidates a valuable asset to retain a poorly performing one (literally throwing good money after bad). It further provides a rationale for the existence of not only DSC ratio default provisions that serve as credible commitments to refrain from cross-subsidization, but also for restrictive call provisions. Interestingly, our model suggests that, in this case, call restrictions arise not from concerns about prepayment as a function of changes in the baseline interest rate, but rather due to changes in the conditional credit spread. That is, a well performing asset (whose credit spread is high, and hence expensive, relative to current default risk) prepays in order to subsidize the continuation of a poorly performing asset (whose credit spread is low, and hence cheap, relative to current default risk).

In the second case of S-NR debt, we quantify that magnitude of the wealth transfer from equity to the in-place debt, showing that it depends critically on cash flow diversification effects and asset investment scale. In particular, we show that cash flow diversification is detrimental to equitholders with respect to the magnitude of the wealth transfer. This follows because it is when the in-place asset is performing poorly and the acquired asset is performing well (resulting from lower cash flow correlation) that leads to wealth transfer and hence underinvestment. If instead both assets are performing poorly at the same time there is nothing for the in-place debtholder to extract the equityholder, and when both assets are performing well there is no financial distress. A further disruptive effect follows from the fact that financial distress on the in-place debt can disrupt the new debt structure, leading to an early liquidation of the newly acquired asset when it is performing well. This disruption amounts to a bankruptcy-induced early call on the S-NR, even when there is no call provision in place. If anticipated by the S-NR lender at the time of issuance, the loan structure is modified to include a credit-based call premium. This restructuring leads to worse execution, causing us to conclude that UN and S-NR debt are not terribly compatible within the same capital structure.

To economize on space we will skip discussion of the third, J-UN case, and focus on the last case. Given PP-UN debt issuance to fund new investment, we show that, even when distinct UN debt sources have equal standing with respect to priority, and when the in-place debt is valued at par at the time of new investment, a wealth transfer nevertheless occurs. This is even though the firm obtains greater loan proceeds at a lower cost relative to financing with NR-BR debt (which realizes no wealth transfer). This surprising relation illustrates the subtle costs and benefits associated with alternative debt funding structures, and in this case follows from cash flow diversification effects and investment scale. Although cash flow diversification reduces the variance in joint asset payoffs, thus causing a reduction in the credit risk of the aggregated PP-UN debt, the benefits of credit risk reduction are shared between the in-place debtholder and the equityholder in proportion to the scale of new investment. That is, increased cash flow diversification benefits both the equity and the in-place debt, where the proportional benefit to the in-place debtholder increases as investment scale declines. This case illustrates that new debt issuance of this type is a Faustian bargain, where, relative to the NR-BR debt issuance option, wealth transfer costs are balanced against increased loan proceeds and a reduced cost of debt.

Lastly, we analyze the GGP bankruptcy through the lens of our model. The events leading up to the Chapter 11 bankruptcy filing, along with events that occurred during the bankruptcy, illustrate many of the managerial objectives and tensions we highlight—most particularly as they apply to the use of non-recourse bankruptcy remote (NR-BR) secured debt to finance asset investment. In response to GGP's Chapter 11 bankruptcy filing that occurred in April 2009, the bankruptcy judge consolidated and recharacterized the NR-BR secured debt issued by GGP. Recharacterization of the NR-BR debt occurred in three primary ways: 1) Suspension of cash trap and reserve accounts; 2) Creation of a centralized cash management account; 3) Dividend payout suspension. These mechanisms caused equity payoffs generated by GGP's performing assets to remain within the firm rather than get distributed to shareholders as dividends, where, according to Paglieri (2011), the retained equity was "up-streamed from the project-level entities to a central cash asset which operated for the benefit of the entire enterprise."

The GGP case illustrates that when NR-BR debt is recharacterized in a bankruptcy proceeding into something more resembling standard secured non-recourse debt, the unique benefits associated with bankruptcy remoteness may be lost at the very time when they are most critical. NR-BR debt recharacterization in the name of a Chapter 11 bankruptcy restructuring may of course make good economic sense if and when the firm's going concern value—something that is uniquely ascribed to the firm and its in-place management, and which generates value to society—would be lost if liquidation would have otherwise occurred without the recharacterization. Our analysis indicates there was little if any management specific going concern value that can be pinned to preserving GGP as a going concern—even during the aftermath of the worst financial crisis to occur in the US in 70 years—leading us to conclude that NR-BR secured debt is appropriate to debt financing commercial real estate assets precisely *because* these assets are not management specific and typically create little or no incremental value that is specific to the firm.

II. Basic Model Structure

II.A. Fundamental Risks, Cash Flows and Asset Value

Throughout the paper, we assume time is discrete, all agents are risk neutral and the riskless rate of interest is zero. There is also an infinite investment horizon, with a realized investment life subject to truncation risk, as will be made clear shortly.

We initially consider a limited liability firm with one asset or group of assets characterized by a single relevant set of parameter values (the "asset"). The asset generates a periodic cash flow net of operating costs, denoted as CF. The cash flow is received at the end of each period, and is vulnerable to two sources of risk. In periods 1 and 2, cash flows experience shocks that proportionally increase or decrease the starting period CF by a factor σ, $\sigma>0$. After the end of period 2, cash flow risk is fully resolved and a second type of risk is introduced. This risk is the probability that the asset, for physical, functional or technological reasons, becomes obsolete by the end of the next period. When this occurs, cash flow production is completely and permanently terminated. Obsolescence probabilities, which we denote as δ, are constant over time and memoryless, implying the probability that the asset is still productive n periods into the future is $(1-\delta)^n$.

Figure 1 visually displays the evolution of cash flows over time. There is no cash flow at t=0. At t=1 cash flow equals either CF$(1+\sigma)$ or CF$(1-\sigma)$, and at t=2 cash flow equals either CF$(1+\sigma)^2$, CF$(1+\sigma)(1-\sigma)$ or CF$(1-\sigma)^2$. Cash flow remains at the time 2 value thereafter, subject to the obsolescence risk described above.

FIGURE 1 HERE

Asset value at various points in time, conditional on particular cash flow realizations, can be calculated as a function of its expected future cash flows. Time t=0 asset value is determined by working backwards in time, starting just after the conclusion of period 2 and at the beginning of period 3. Given the periodic obsolescence risk summarized by the parameter, δ, and recalling that all agents are risk neutral and the risk-free rate is zero, a value factor of $\frac{1-\delta}{\delta}$ results that transforms the starting period cash flow into an asset value. This value factor is seen to be decreasing in the risk of obsolescence, going from $\infty \rightarrow 0$ as δ goes from $0 \rightarrow 1$.

With this present value factor, the asset's t=2 value, inclusive of cash flow receipt at that time, can be determined. The t=2 value is conditional on one of three possible up (U) versus down (D) cash flow realizations: U|U, U|D^D|U, or D|D (where ^ denotes "or" – also see Figure 1). Conditional asset values are as follows:

<u>t=2</u>

$$V_2^{U|U} = CF(1+\sigma)^2 + \frac{CF(1+\sigma)^2(1-\delta)}{\delta} = \frac{CF(1+\sigma)^2}{\delta} \tag{1a}$$

$$V_2^{U|D} = V_2^{D|U} = CF(1+\sigma)(1-\sigma) + \frac{CF(1+\sigma)(1-\sigma)(1-\delta)}{\delta} = \frac{CF(1+\sigma)(1-\sigma)}{\delta} \tag{1b}$$

$$V_2^{D|D} = CF(1-\sigma)^2 + \frac{CF(1-\sigma)^2(1-\delta)}{\delta} = \frac{CF(1-\sigma)^2}{\delta} \tag{1c}$$

With these t=2 conditional asset values, we are now in a position to calculate asset values at t=1. Doing so requires probabilities associated with U versus D realizations. Using standard binomial valuation results therefore based on imposing the law of one price, and again recalling agent risk neutrality, a zero risk-free rate and symmetric value changes, the endogenous probability of a U versus D state outcome is ½. With this, the t=1 conditional asset values inclusive of the end-of-period cash flows are:

<u>t=1</u>

$$V_1^U = CF(1+\sigma) + \left(\frac{1}{2}\right)\frac{CF(1+\sigma)}{\delta}[1+\sigma+1-\sigma] = \frac{CF(1+\sigma)(1+\delta)}{\delta} \tag{2a}$$

$$V_1^D = CF(1-\sigma) + \left(\frac{1}{2}\right)\frac{CF(1-\sigma)}{\delta}[1+\sigma+1-\sigma] = \frac{CF(1-\sigma)(1+\delta)}{\delta} \tag{2b}$$

The time t=0 asset value can now be calculated. In doing so, keep in mind there is no cash flow received at that time, implying that asset value is simply the expected present value of t=1 asset values:

<u>t=0</u>

$$V_0 = \left(\frac{1}{2}\right)\frac{CF(1+\delta)}{\delta}[1+\sigma+1-\sigma] = \frac{CF(1+\delta)}{\delta} \tag{3}$$

Figure 2 presents a visual depiction of the evolution of asset values. We note that, based on our modeling assumptions, t=0 asset value, V_0, depends only on the asset's productivity (CF) and obsolescence risk (δ), and not on the near-term symmetric cash flow shock risk as summarized by σ. If instead a positive risk-free rate were introduced, V_0 would depend on that rate as well as σ. Risk aversion would also cause V_0 to depend on σ. We further note that our assumption of two non-overlapping sources of risk is made for convenience in order to simplify the modeling. As we will shall see, this asset risk decomposition greatly simplifies the analysis while nevertheless allowing for a novel as well as rich characterization of asset valuation.

FIGURE 2 HERE

In reference to equation (3), note the obsolescence risk parameter, δ, acts like a real capitalization rate applied to translate current cash flow into asset value. As a cap rate measure, it does, however, work in a reverse direction from the way it is usually expressed (see Bokhari and Geltner (2015)). The interpretation is as follows. After controlling for the appropriate nominal cash flow discount rate and the anticipated nominal cash flow growth rate due to inflation, the residual component to cash flow growth has two components: smoothed changes in real cash flow growth resulting from changes in fundamental space demand and real declines in cash flow growth due to physical depreciation and economic obsolescence risk. The obsolescence risk parameter from our model captures the latter component in the residual, where obsolescence risk is also informed by the former component.

For example, an office building in midtown Manhattan will, for two related reasons, typically experience lower obsolescence risk than an office building in suburban St. Louis. First, barring a major unforeseen catastrophic event, because it is midtown Manhattan, there is little chance that land and office building in midtown Manhattan will experience a major shock that renders the asset completely worthless. If office space would somehow experience a major negative demand shock, the land underneath the building, and likely the building itself, would nevertheless retain significant value. Second, land values in midtown Manhattan are a high percentage of building value, the latter of which physically depreciates while the former does not. The same things cannot be said for suburban St. Louis office property, implying lower δ's and higher valuations for midtown Manhattan office property versus higher δ's and lower valuations for suburban St. Louis office property.

Both sources of risk in our model—cash flow uncertainty and obsolescence risk—can be used to characterize different types of assets. To extend our previous example, in a commercial real estate (CRE) context, centrally located assets in high-barrier, high demand markets like NYC and San Francisco, will have a low risk of obsolescence, but potentially significant near-term cash flow volatility. Lower quality retail properties and other faddish retail formats may experience a high risk of obsolescence, whereas near-term cash flow risk may be moderate. A single-tenant property with a high credit quality tenant and a long-term lease will experience relatively low cash flow and obsolescence risk, whereas the opposite may be true with a low credit quality tenant.[2]

II.B. Debt Financing

We now consider the debt financing of the asset characterized above. Debt is the preferred funding mechanism for the firm due to significant contracting costs associated with outside equity as it relates to high state verification costs, as well as a scarcity of inside equity. The high cost of both outside and inside equity causes firm insiders to ration that their own equity as much as possible across existing and anticipated future investments. In a similar vein, firm insiders distribute all residual cash flows as dividends rather than hold that cash idle inside the firm. As a result, we say the firm is financially constrained, where, all else equal, firm managers prefer to issue as much debt as possible to fund investment.

The resolution of firm-specific cash flow uncertainty at the end of two periods and the introduction of obsolescence risk after that point makes it natural to focus on debt with a 2-period maturity.[3] Then, conditional on the firm maintaining control of the asset at debt maturity (i.e., not defaulting on the debt on or prior to debt maturity), the firm can fund debt repayment through either an asset sale or a debt refinancing. In the case of debt refinancing, it would be the case that the firm pledges its future stream of expected cash flows in order to maximize new debt issuance proceeds. A credit spread is required to compensate the debtholder for the risk of obsolescence, which abruptly terminates the cash flow stream and causes a complete writeoff of the loan balance. The following lemma states the resulting loan amount and implied credit spread.[4]

<u>Lemma 1</u>*: Suppose the firm has paid off its existing debt and controls the asset at the end of period 2. Let* $CF_2^{\#|\#}$ *denote the cash flow realized at the end of that period conditional on the 2-period up versus down state outcomes, #|#. The resulting ex-dividend asset value is* $\frac{CF_2^{\#|\#}(1-\delta)}{\delta}$ *after accounting for exposure to obsolescence risk. Total proceeds from an asset sale or debt refinancing equal the time t=2 ex-dividend asset value. In the*

[2] Other firms such as technology licensing enterprises may own assets with low cash flow shock risk while also being subject to high obsolescence risk. In contrast, infrastructure companies may be subject to highly variable demand with little risk of obsolescence, thus experiencing the exact reverse set of risks. Firms with other risk combinations are easy to imagine and therefore characterize with the pair (σ,δ).
[3] We will not consider the issue of optimal debt maturity in this paper, but with minimal additional model supplementation it would be straightforward to endogenously generate the debt maturity structures identified herein.
[4] All proofs will be added later in an appendix.

case of debt financing, the debt term is perpetual, the periodic debt payment is $CF_2^{\#|\#}$*, and the implied credit spread on the debt is* $\frac{\delta}{1-\delta}$.

The implied credit spread of $\frac{\delta}{1-\delta}$ is increasing in the obsolescence risk parameter, δ. The existence of obsolescence risk without the presence of material, firm-specific cash flow uncertainly, lends itself to long-term debt financing. We note that this type of perpetual, fixed-payment debt would not, however, work should cash flows decline at a constant and certain rate of δ over time instead of being exposed to the risk of sudden death. The gradual as opposed to sudden death of the asset, with its declining cash flow stream, would instead require debt payments to decline over time, in effect requiring the constant refinancing of the debt at one-period terms to maturity.

Now consider debt financing at time *t*=0. As noted, the debt will mature in two periods in anticipation of either an asset sale or takeout debt financing as described in Lemma 1. The debt is fixed payment, with a coupon interest payment required at times *t*=1,2, and the restriction that the coupon interest rate is set so that the debt is issued at par. Par valuation at issuance is imposed to simulate industry practice and aid in comparing across financing alternatives.

To eliminate any possibility of default, loan issuance proceeds could be limited so that sufficient cash flow and asset value existed at *t*=2 to fully fund debt payment in a D|D state of the world. In this case the debt is risk-free with coupon payments of zero. In this case, with the firm's objective of maximizing issuance proceeds, the time *t*=0 debt proceeds and *t*=2 debt repayment amount would equal $V_2^{D|D} = \frac{CF(1-\sigma)^2}{\delta}$, as in equation (1c). Given an initial asset value of $V_0 = \frac{CF(1+\delta)}{\delta}$ as reported in equation (3), the *t*=0 debt-to-firm value ratio is $LTV^{RF} = \frac{(1-\sigma)^2}{1+\delta} < 1$. As intuition would suggest, this ratio is decreasing in both cash flow uncertainty, σ, and the risk of asset obsolescence, δ.

Although this case of riskless debt financing has certain interesting characteristics and implications, it fails in one important regard—which is to relax financial constraints of firm insiders as much as possible. High shadow costs of inside and outside equity cause firm managers to want to maximize loan issuance proceeds at the expense of possible default and loss of control. This in turn focuses attention on the lender, who is put into a position to impose limits on credit risk and hence debt proceeds in order to comply with internal and/or regulatory risk-bearing constraints. We will label the underwriting risk limits as lender participation constraints (LPC), which impose a maximum loan amount as L and a maximum coupon payment as ιL. The LPC's are as follows[5]:

$$\underline{\text{LPC1}}:\ \iota L + L \leq \frac{CF(1+\sigma)(1-\sigma)}{\delta} \tag{4a}$$

$$\underline{\text{LPC2}}:\ \iota L \leq CF(1+\sigma) \tag{4b}$$

The first participation constraint stated in (4a) requires sufficient cash flow and residual asset value to fund full repayment of debt interest and principal given a U|D^D|U realization at t=2. This implies term default conditional on reaching the D|D node at t=2, which in turn implies that $\iota L > 0$. In other words, the credit spread is positive given the potential for loan default. The second participation constraint requires sufficient cash flow from the asset in place given a U state at t=1 to fund the coupon payment; otherwise, sure cash flow default would occur at t=1 to undermine the loan structure. This constraint implies the lender is willing to entertain the possibility of liquidity-based cash flow default at the D node at t=1. Should this second constraint (4b) bind, it will require reducing loan proceeds so that LPC1 is no longer binding.

In all cases, given the firms objective of maximizing debt proceeds subject to the lender's debt underwriting constraints, either LPC1 or LPC2, or conceivably both, will be binding for any given set of feasible parameter values. As such, and given the restriction that lenders make zero profits in competitive equilibrium, the coupon payment, ιL, and the loan amount, L, are endogenously determined. This in turn generates endogenously determined debt-to-firm value ($LTV=L/V_0$) and debt service coverage ($DSC=CF/\iota L$) ratios that vary depending on the asset's fundamental productivity and risk characteristics.

[5] Implicit in these constraints is a dividend payout restriction, which requires the firm to fund the contractually stated coupon payment and any loan principal payoff prior to distributing operating cash flows as dividends. However, once the dividend payout has occurred, there is no legal recourse to prior dividend payouts should contract terms be violated due to default.These payout restrictions are standard. With project lending the dividend restriction is often referred to as a "lockbox" requirement. Recourse and possible clawback of previously paid dividends is also sometimes allowed when certain "bad acts" such as fraud or misrepresentation occur. Clawbacks are not considered here. Finally, lenders sometimes require the funding of certain reserves such as capital expenditures prior to approving dividend payouts. We will also not consider funding reserves of this type in the formal model, but we will touch on the issue again in section VI in our analysis of the GGP bankruptcy.

III. Characteristics of Risk Debt Funding Structure: The Single Asset Case

In this section we characterize the debt funding structure as it depends on satisfying participation and/or incentive compatibility conditions of both the lender and the borrower. As previously stated, the firm's primary objective is to maximize debt proceeds subject to satisfying lender underwriting participation constraints. Conditional on that, with the further objective of minimizing post-issuance debt value, the borrower's optimal repayment-default decisions are made conditional on state outcomes at times t=1, 2. The lender rationally anticipates the set of feasible-optimal borrower actions along the path of potential state outcomes in determining a loan quantity (loan proceeds, L) and loan price (credit spread, ι) in Bayesian-Nash equilibrium.

Our solution strategy is to consider alternative debt funding structures as they depend on whether LPC1 or LPC2 is binding, and then verify borrower participation (liquidity) and incentive compatibility (strategically optimal) conditions necessary for the funding structure to work. There are three major cases to consider in total, with two sub-cases embedded in the third case. In the first case, LPC1 is binding and the loan's coupon payment, ιL, is sufficiently small so as to ensure there is no cash flow-liquidity default loan default at t=1. As a result, there is only a *term default* at the D|D node at t=2. In this case, borrower funding of the coupon payment at t=1 is incentive compatible as well as financially feasible. In the second case, LPC1 is binding, but there is insufficient cash flow given a D outcome at t=1 to fund the coupon payment. This *early term default* case is a liquidity default, but it is also incentive compatible, meaning the borrower has no incentives to source outside funds in an effort to bridge the funding shortfall. A third case, occurs when LPC2 is binding, implying a reduction in loan proceeds is required so as to avoid a cash flow default at the U node at t=1. In this case, an early term default nevertheless occurs at the D node at t=1 due to insufficient cash flow to fund coupon payment. The scaling back of loan proceeds in this case changes payoffs to equity conditional on loan repayment at t=2, which potentially modifies the borrower's incentives to fund coupon payment at the D node at t=1. It is this potential change in incentives that generates the two subcases noted previously.

The lender make zero expected profits. Satisfying the lender's underwriting participation constraints, as well as the borrower's participation (liquidity) and incentive compatibility (optimality) conditions, generates endogenously determined coupon payment, ιL, and loan proceeds, L. The following proposition summarizes lending outcomes for the three cases noted above:

Proposition 1: Three distinct lending regimes occur depending on cash flow uncertainty, σ, and the risk of asset obsolescence, δ:

Regime 1 is characterized by a binding (non-binding) LPC1 (LPC2) constraint, with no cash flow default at t=1 and term default at the D|D node at t=2. This case is realized when $\delta \geq \frac{\sigma}{4}$. In this case, $L_1 = \frac{CF(1-\sigma)}{\delta}\left[1+\frac{3\sigma}{4}\right]$ and $\iota_1 L_1 = \frac{CF\sigma(1-\sigma)}{4\delta}$. Coupon payment at the D node at t=1 is both financially feasible and incentive compatible. L_1 (loan quantity) is decreasing in both σ and δ, and ι_1 (loan price) is increasing in σ but is insensitive to δ.

Regime 2 (C2) is characterized by a binding (non-binding) LPC1 (LPC2) constraint, with coupon payment default at the D node at t=1. This case is realized when $\frac{\sigma(1-\sigma)}{4+2\sigma} \leq \delta < \frac{\sigma}{4}$. In this case, $L_2 = \frac{CF(1-\sigma)}{\delta}\left[1+\frac{2\sigma+\delta}{3}\right]$ and $\iota_2 L_2 = \frac{CF(\sigma-\delta)(1-\sigma)}{3\delta}$. Coupon payment default at the D node at t=1 occurs because the coupon payment exceeds available cash flow, where non-payment is also incentive compatible. L_2 is decreasing in both σ and δ, and ι_2 is increasing in σ but decreasing in δ.

Regime 3 is characterized by a non-binding (binding) LPC1 (LPC2) constraint, with scaled back loan proceeds to avoid coupon payment default at the U node at t=1. Coupon payment default occurs at the D node at t=1. This case is realized when $\delta < \frac{\sigma(1-\sigma)}{4+2\sigma}$. In this case, $L_3 = \frac{CF(1-\sigma)}{\delta}\left[1+\frac{\delta(3+\sigma)}{(1-\sigma)}\right]$ and $\iota_3 L_3 = CF(1+\sigma)$. L_3 is decreasing in both σ and δ, and ι_3 is increasing in σ and δ. Within this region, when $\delta < \frac{\sigma(1-\sigma)}{4+6\sigma}$, non-payment at the D node at t=1 is, however, no longer incentive compatible, although full coupon payment remains financially infeasible. The existing loan structure is robust to this sub-case as long as the lender believes the borrower cannot or will not bridge the coupon payment funding gap.

In all three cases, loan proceeds exceed those obtained when the loan is riskless, where $L_1 \leq L_2 \leq L_3$. Loan amounts and coupon payments are continuous over the entire feasible domains of σ and δ, and are continuously differentiable (smooth) except at the regime switching points of $\delta = \frac{\sigma}{4}$ and $\delta = \frac{\sigma(1-\sigma)}{4+2\sigma}$.

The model generates a rich set of results, with three "lending regimes" that depend on the size of *δ* relative *σ*. We will refer to the regimes as high, moderate and low obsolescence risk (*δ*), or real "cap rate" regions, appealing also to Figure 3 as we go along.

FIGURE 3 HERE

Regime 1 follows when cash flows are large relative to the asset value; i.e., when the real cap rate is high (see Panel A of Figure 3). The lower relative asset value implies that the LTV (LPC1) constraint is binding while the cash flow-based debt service coverage ratio (LPC2) is not. Given the binding LTV underwriting constraint, loan amount increases within the region as the cap rate declines (see Panel A of Figure 3). The loan amount also increases as cash flow shock uncertainty is reduced, since such a reduction also increases t=2 asset value at the LTV underwriting constraint. Interestingly, the credit spread (ι) does not depend on the cap rate in this region. This follows because there is no cash flow default in this region, implying the loan coupon payment expands proportionally with loan amount to leave ι unaffected.

The moderate cap rate/asset value region is such that cash flows are of moderate size relative to asset value (see Panel B of Figure 3). In this region there is cash flow default at the D node at t=1, with a binding LTV underwriting constraint. Liquidity default is incentive compatible at the D node at t=1, since there is no residual equity value from continuation, implying the strategic default decision comes down to whether current cash flow exceeds the coupon payment or not (which it doesn't). The combination of a binding LTV underwriting constraint and cash flow default at t=1 suggests that both σ and δ should impact the credit spread, which they do. However, somewhat unintuitively, increasing the cap rate through δ has the effect of decreasing the credit spread. This happens because the increase in cap rate reduces the loan amount due to the binding LTV underwriting constraint, while at the same time cash flow increases in proportion to asset value. This proportional increase in cash flow benefits the lender since it reduces the gap between coupon payment amount and cash flow at the D node at t=1 to reduce losses, resulting in an decrease in the credit spread.

Region 3 follows from a low cap rate, implying high asset valuation such that cash flows are small relative to asset value (see Panel C of Figure 3). In this case the cash flow underwriting constraint (LPC2) binds, which follows given the relatively small cash flows that exist to support the borrower's objective of maximizing debt proceeds. Loan amount increases with a lower cap rate, since, holding cash flow constant, the value of the asset increases to increase collateral recovery in default. Here the credit spread also decreases as the cap rate decreases, for basically the same reason that explains why the loan amount increases. The comparative static relation with respect to δ flips, however, in going from region (C2) to (C3). This happens because the loan goes from a binding LTV underwriting constraint (LPC1) to a binding debt service coverage underwriting constraint (LPC2).

Finally, when the real cap rate is sufficiently small within regime 3, resulting in a very high asset value relative to cash flow, it is no longer incentive compatible for the borrower to default at the D node at t=2. This happens even though cash flow is insufficient to fund the debt coupon payment, and follows because sufficient residual equity value now exists at the U|D^D|U node at *t*=2, so that continuation is optimal at the D node at *t*=1. The integrity of the loan structure is maintained if the lender believes the borrower cannot or will not bridge the coupon payment funding gap with an outside equity contribution. However, if the lender modifies its belief such that the firm is willing and able to fund the coupon payment shortfall when a D realization occurs at t=1, the loan structure is modified to account for this belief. The following corollary lays out the technical details for the modified loan structure.

Corollary 1 to Proposition 1: Assume $\delta < \frac{\sigma(1-\sigma)}{4+6\sigma}$ so that continuation at the D node at t=1 is incentive compatible. Further, suppose that the lender believes the firm is willing and able to bridge the coupon payment cash shortfall given a D realization at t=1 with a liquidity infusion coming from outside the firm. Then, in anticipation of this possible outcome, the loan structure changes from the stated loan structure in Proposition 1, where now $L_3^{\#} = \frac{CF(1-\sigma)}{\delta}\left[(1-\sigma) + \frac{7\delta(1+\sigma)}{1-\sigma}\right] < L_3$, implying the credit spread increases relative to ι_3. The revised loan structure endogenously changes the firm's incentives to fund the coupon payment gap, where now continuation is incentive compatible only when $\delta < \frac{\sigma(1-\sigma)}{8+12\sigma}$.

The key to understanding this result is to recognize that the firm increases its own equity value by funding the coupon payment to avoid a liquidity default. This happens because a liquidity default *increases* the loan value relative to a no-default outcome, and even though the asset value is underwater relative to the loan's par value. When the lender anticipates an equity infusion to cover the coupon payment gap, the lender modifies the initial loan structure by reducing loan proceeds and increasing the credit spread. As a result of the modified loan terms, the continuation option is lost under the revised loan structure when $\frac{\sigma(1-\sigma)}{8+12\sigma} < \delta < \frac{\sigma(1-\sigma)}{4+6\sigma}$. In this range, liquidity default occurs to the benefit of the lender, resulting in not only inferior loan execution for the borrower but also loan costs that exceed the fair cost of the loan.

We note that our model suggests this situation occurs only in the case of very high value assets, creating a conundrum for the borrower due to the optionality associated with the funding decision. To resolve this problem, a debt service coverage default provision can be introduced into the debt contract that is capable to maintaining the integrity of the original loan structure. This provision, which is triggered when asset cash flow is insufficient to meet the coupon payment amount, limits the discretion of the borrower by giving the lender the right to declare default, accelerate loan repayment, and liquidate the loan when the asset value is insufficient to fully repay the debt. The debt service coverage provision in this subcase thus serves as a commitment device, added to the debt contract at the behest of the borrower, that allows the firm to obtain *better* loan terms—higher debt issuance proceeds and a lower credit spread—than it might otherwise be obtainable.

This provides a new rationale for debt service coverage provisions included in debt contracts, which function as incentive compatibility mechanisms that limit borrower discretion to fund coupon loan payments with outside liquidity when cash flows and asset values deteriorate. The presence of such provisions are predicted to be associated with high-value assets with low relative cash flows. Low cash flows increase the likelihood of liquidity default when the borrower would otherwise like to continue to fund payment to retain control of its high-value asset.

Our last task in this section is to more fully describe the debt's structural characteristics in the context of the firm's liquidity constraints and the lender's underwriting constraints. As outlined in Proposition 1, lender underwriting participation constraints endogenously affect both the loan issuance proceeds, *L*, and the coupon payment, ιL, which in turn affect not only the firm's *LTV* ratio but also the firm's debt service coverage ratio, which we denote as $DCR = \frac{CF}{\iota L}$. The following corollary considers *LTV* and *DCR* as they vary within and across the various underwriting-lending regimes outlined in Proposition 1.

Corollary 2 to Proposition 1: Define $LTV_j = \frac{L_j}{V_0}$ *and* $DCR_j = \frac{CF}{\iota_j L_j}$, *j=1,2,3 indicating the three lending regimes previously identified. When* $\delta \geq \frac{\sigma}{4}$ *(Regime 1, LPC1 binding),* $LTV_1 = \frac{(1-\sigma)}{1+\delta}\left[1 + \frac{3\sigma}{4}\right]$ *and* $DCR_1 = \frac{4\delta}{\sigma(1-\sigma)}$. LTV_1 *is decreasing in both σ and δ, and* DCR_1 *is increasing in δ but is decreasing in σ. When* $\frac{\sigma(1-\sigma)}{4+2\sigma} \leq \delta < \frac{\sigma}{4}$ *(Regime 2, LPC1 binding),* $LTV_2 = \frac{(1-\sigma)}{1+\delta}\left[1 + \frac{2\sigma+\delta}{3}\right]$ *and* $DCR_2 = \frac{3\delta}{(1-\sigma)(\sigma-\delta)}$. LTV_2 *is decreasing in both σ and δ, and* DCR_2 *is increasing in δ but is decreasing in σ. When* $\delta < \frac{\sigma(1-\sigma)}{4+2\sigma}$ *(Regime 3, LPC2 binding),* $LTV_3 = \frac{(1-\sigma)}{1+\delta}\left[1 + \frac{\delta(3+\sigma)}{(1-\sigma)}\right]$ *and* $DCR_3 = \frac{1}{1+\sigma}$. LTV_3 *is decreasing in σ but is increasing in δ, and* DCR_3 *is decreasing in σ but is insensitive to δ. The maximum LTV,* LTV^{Max}, *is realized when* $\delta = \frac{\sigma(1-\sigma)}{4+2\sigma}$, *where* $LTV^{Max} = \frac{\left(\frac{1-\sigma}{3}\right)[(3+2\sigma)(4+2\sigma)+\sigma(1-\sigma)]}{4+2\sigma+\sigma(1-\sigma)} < 1$, *occurring at the boundary between regimes 2 and 3.*

It is important to keep in mind that this analysis is most applicable to secured non-recourse secured debt, due to the fact the analysis occurs on a "stand-alone" basis. The LTV ratio increases as the real cap rate decreases in regions 1 and 2, which follows from the binding LTV underwriting constraint and the fact that asset value is directly decreasing in the real cap rate. The LTV ratio declines as the real cap rate declines in region 3, however. This follows because the DCR is now the binding underwriting constraint, which scales back the loan amount relative to the asset's value. The fact that DCR is decreasing in σ follows from the fact that the credit spread increases rapidly for increases in σ, which results in a net increase in the required coupon payment. In contrast, the increase in the DCR for increases in δ follow from reductions in the coupon payment. In other words, δ is more closely tied to the loan amount as it depends on asset value, whereas σ operates more directly through the coupon payment due its adverse affect on the credit spread.

Although our model generates a number of empirical predictions, perhaps the most interesting have to do with the behavior of the LTV and credit spread on secured non-recourse debt as they depend on the real cap rate. Consider, for example, office property located in Manhattan (NYC) versus suburban St. Louis (SSL). NYC property generates low cash flow relative to asset value, whereas the opposite is true in SSL. There is no clear prediction regarding LTV's in levels in NYC versus SSL, where cash flow uncertainty as summarized by σ will be a determining factor. The model, however, generates distinct predictions at the margin. LTV is predicted to decrease in the real cap rate in SSL, due to the drop in collateral asset value as the determining factor in the loan proceeds. This is not the case in NYC, because the constraint on loan proceeds relates to cash flow and not asset value. As for credit spreads, there again is no clear prediction regarding levels as a function of the real cap rate, where again the relation will depend on, among other things, relative cash flow uncertainty in the short term. But, at the intensive margin, credit spreads are predicted to be insensitive changes in real cap rates in SSL. The insensitivity of credit spreads in SSL follows from the already low loan amount relative to cash flow, which continues to support the integrity of the loan structure even though real cap rates have increased. In contrast, credit spreads are highly sensitive to changes to cap rates in NYC, where an increase in the relatively low cap rate in NYC is predicted to increase the credit spread on debt. This happens because the increase in cap rate from a low level has a large impact on asset value, where, at the same time, cash flow is low relative to asset value. The low relative cash flow decreases recovery in default to increase lender credit risk.

IV. New Investment and Debt Funding Choice

IV.A. General Framework and Debt Priority Structure

In this section we move on to consider a firm which has an asset in place that is subject to cash flow uncertainty and obsolescence risk as described in section II. This asset is underwritten and financed with debt of the type described in sections II and III in which lender participation constraints LPC1 and LPC2 apply. As of current time t=0, the debt is priced at par with maturity in two periods, as previously described. Unless otherwise noted, as a baseline we assume this debt is underwritten, priced and issued without explicit anticipation of the arrival any new investment opportunities and debt issuances.

Now assume that at time $t=0^+$ the firm is presented with a new investment opportunity. This investment opportunity comes in the form of an asset that generates cash flows exposed to the cash flow uncertainty and obsolescence risks described previously. To simplify and focus the analysis, we will assume that new investment cash flows are risk-identical to the asset in place. By this we mean that the cash flow shock uncertainty parameter, σ, and the obsolescence risk parameter, δ, are common for the two assets.

The new investment does differ from the asset in place along two critical margins, however. First, the scale of the new investment is such that it is the same size or smaller than the asset in place, where we introduce the parameter, α, $0<\alpha\leq 1$, to quantify the scale effect. Second, we allow for there to be imperfect correlation between cash flows generated by the asset in place and the new investment, where we denote cash flow correlation with the parameter, ρ. Imperfect cash flow correlation applies during the two initial periods during which cash flow uncertainty risk predominates.[6]

For similar reasons as described in section II, we will assume inside equityholders are motivated to issue as much debt as they can to finance the new investment. Previously we did not specifically identify whether the debt was secured or unsecured, as debt priority in the single-asset limited liability firm in which there is only one source of debt and equity is not a primary concern. Now, however, we consider new investment and the financing of that new investment with debt. The current framework implies a more complex asset and capital structure, where now debt priority structure in this dynamic, multi-asset firm is central to the analysis.

Unless otherwise noted, we will assume that the in-place debt is unsecured (UN). It will become apparent that this form of debt allows for maximum flexibility in modifying the firm's capital structure to accommodate other forms of debt that vary based on their relative priority structure, and hence the payoffs to

[6] After second period cash flows are realized, for our immediate purposes it does not matter whether obsolescence risk is correlated or not, since t=2 asset values are, both individually and collectively, identical regardless of how obsolescence risk might be correlated between the two assets.

debt and equity (also see Stulz and Johnson (1985)). Given the in-place debt is unsecured, to finance the new investment we will consider four alternative debt funding structures that vary in their security and hence their priorities: 1) Non-recourse, bankruptcy remote secured debt (NR-BR); 2) Standard non-recourse secured debt (S-NR); 3) Junior unsecured debt (J-UN); and 4) *Pari passu* unsecured debt (PP-UN). Each of these debt structures has unique characteristics which we exploit in order to highlight implications of claim priority in the multi-asset firm, and how inefficiencies associated with underinvestment incentives are and are not addressed through debt funding decisions.

These differing characteristics are spelled out in detail as we proceed through this and the following sections. At this point, however, we do want to distinguish between NR-BR and S-NR secured debt, since differences between the two debt forms has mostly either been ignored or muddled in the literature.[7] These two debt forms are similar in their non-recourse and security features, which limits the debtholders' ability to recovery anything beyond the pledged collateral in default-liquidation. This creates something of a wall between the debt and the assets securing the debt, and other debt and assets in the firm. The wall is impenetrable from the inside going out. But, in the case of S-NR debt, the wall is not impenetrable from the outside going in, meaning that external debtholders can extract available equity or possibly disrupt payoffs to the S-NR debtholders if external debt provisions are violated through payment default or for some other reason.

The detailed characteristics of NR-BR secured debt bear additional discussion. The non-recourse aspect of the debt is clear: the debtholder has no recourse to any security other than that which has been designated in the loan contract. Bankruptcy remoteness is further intended to prevent claimholders from other parts of the firm from reaching over to into the structure to take away any payoffs to debt or equity. This implies that any excess cash be paid out to the relevant equityholders within the structure, where neither the NR-BR debtholder nor any other claimholders in the firm has the legal authority to claw back those dividends. If, alternatively, there were requirements that the relevant dividends stay within the firm in a cash account, with firm managers having the authority distribute that cash as it sees fit to assets or financial claimholders outside the NR-BR debt structure, such a cash retention structure defeats the original intent of the NR-BR debt structure.

[7] There has been a clearer delineation the legal literature, where there has been a recent focus on bankruptcy remote debt structures as a mechanism to "opt out" of the Chapter 11 bankruptcy process.

Thus, a subtle feature of NR-BR debt is that all excess cash is effectively paid out to the relevant equityholders. The NR-BR debt structure does not, however, in general prevent the relevant equityholders from infusing equity back into the structure should they decide it is in their interest to do so. Discretion along these lines would seem innocuous, since bankruptcy remoteness prevents claimholders from outside the structure from grabbing the equity, and an equity infusion would intuitively seem to never detract from debt value. This is not always the case, however, as shown in proposition 1 and corollary 1 to the proposition, where an equity injection occurs due to the fact that liquidity default is not incentive compatible for the borrower. The bridge funding coupon payment actually *decreases* debt value to the detriment of the NR-BR debtholder, the possibility of which destabilizes the debt structure to begin with. This form of "reverse leakage", which violates the notion that NR-BR is "hermitically sealed," requires the addition of other debt provisions if the debtholder wants to prevent such events from occurring.

With these distinctions in mind, we will now state several foundational results that will be useful as we move forward.

Lemma 2: Given the setting described above with two assets and the four alternative debt funding structures, the following relations obtain: 1) The issuance of NR-BR debt isolates the in-place debt, whatever its current form, to transform the in-place debt to NR-BR debt; 2) If S-NR debt replaces the UN debt as the in-place debt, then the issuance of S-NR is invulnerable to equity devaluation; 3) With UN debt in place, only the issuance of NR-BR secured debt is uniquely capable of fully preventing wealth transfers and resolving the underinvestment problem.

The issuance of NR-BR debt secured by specified assets is equivalent to creating a subsidiary firm that is itself limited liability, and thus isolated from other parts of the firm. In particular, this issuance strategy isolates *both* the NR-BR debt and associated equity payoffs from outside influence, and in doing so isolates the debt and equity located in other parts of the firm. Consequently, in its basic form, there are no debt contracting-financial claim valuation externalities, going in either direction, associated with the issuance of NR-BR debt. No other debt structure offers complete separation of payoffs to debt and equity in the multi-asset firm, and is fundamentally the reason why NR-BR debt is uniquely suited to fully resolving the underinvestment.

NR-BR debt is distinguished from S-NR debt in that, although payoffs to the S-NR debtholder are unaffected by what happens in other parts of the firm, payoffs to equity are not. However, if the in-place debt were S-NR debt instead of UN debt, the subsequent issuance S-NR debt is protected from value extraction from existing debtholders. This happens because of the non-recourse feature, which blocks both S-NR debtholders from reaching over to grab available equity value from the other non-secured asset.

The following lemma highlights an important tradeoff that exists between flexibility in assimilating new debt forms into an existing capital structure and underinvestment incentives related to wealth transfers associated with the financing of new investment. We also consider how in-place callable debt can help resolve the costs associated with this flexibility-underinvestment tradeoff.

<u>Lemma 3</u>*: Let UN debt in place be characterized by: i) no reference to specific assets as security, and ii) no covenant specifying the debt's priority relative to other debt forms. Given these characteristics, UN debt in place provides a flexible capital structure in terms of an ability for firm managers to alter payoffs to debt and equity as a result of the financing of new investment. But this flexibility creates significant vulnerability to wealth transfers and underinvestment incentives vis-à-vis new debt contracting externalities. NR-BR debt is the least flexible form of in-place debt. But, the inflexible structure is (nearly) invulnerable to payoff reformulation, implying it can fully resolve the underinvestment problem. When the in-place debt is valued at par or a premium to par, inflexibility and underinvestment can be simultaneously defeated with the inclusion of a call provision allowing the in-place debt to be repaid at par at the time of new investment.*

This lemma is perhaps best understood in the context of the four debt structures we consider. Payoffs to in-place UN debt are easily modified depending on the form of new debt issued. In the case of NR-BR debt issuance, UN is isolated and transformed into NR-BR debt. This happens because the UN debtholder's ability to access or otherwise effect debt or equity payoffs associated with the new investment is blocked by bankruptcy remoteness. In the case of S-NR issuance, although the structure blocks the new debtholder from grabbing equity or debt payoffs associated with the in-place asset, it does not block access to equity value should there be any available. The issuance of J-UN debt to finance the new investment has the potential to significantly improve the value of the in-place UN debt, since the latter now assumes a senior position in the capital structure with the addition of the new asset value. To compensate for the subordinate position in the capital structure, the J-UN debtholder may simultaneously scale back issuance proceeds and increase the credit spread, thus creating significant underinvestment problems for in-place equityholders. Lastly, the issuance of PP-UN debt results in the two distinct debt sources becoming, in effect, one unified debt claim as a result of the cross-collateralization of all the assets in the structure. Payoffs to the in-place UN debtholder change, for example, depending on the scale of the new investment and the correlation structure of the underlying asset cash flows.

The ability to call the existing debt at par when that debt is currently priced at par or a premium to par eliminates inflexibility costs as well as costs to underinvestment. This follows because the call option "wipes clean" the liability structure of the firm at no cost to equity, allowing for the issuance of whatever combination of debt claims is optimal at the time. This result begs the question of why call provisions are not more commonly observed in debt contracts, and why the "start over" call option is not exercised more often by firms. Later we will consider potential explanations.

IV.B. Underwriting Constraints and Cash Flow Correlation Structure

In section V we explore the relations and results stated in lemmas 2 and 3 in detail. To prepare for that analysis, which considers a dynamic firm that undertakes new investment and the financing of that investment, we first need to augment lender underwriting-participation criteria to accommodate the new debt financing.

As noted above, we will assume UN debt that matures in two periods is used to finance the in-place asset, where LPC1 and LPC2 stated in equations (4a) and (4b) are satisfied. Results outlined in proposition 1 apply, where the relation between σ and δ is critical. The new debt will also mature in two periods, for reasons previously discussed. For cases in which the new debt is specifically secured by the new investment (and only the new investment), the issuance of either S-NR or NR-BR debt alleviates debtholder concerns over contracting externalities that could impact potential debt payoffs. As a result, the new debt can be underwritten on a "stand-alone" basis in accordance with the lender participation constraints introduced in section II. However, when the new debt is issued on an unsecured basis, as in the cases of J-UN or PP-UN debt, payoffs to the newly issued debt will generally be affected by the promised payoffs to the in-place debt as they depend on the combined payoffs to the two assets, implying that new debt underwriting criteria recognize these state-contingent payoffs.

We refer to the in-place asset as asset-1 and the newly acquired asset as asset-2. Recalling that asset-2 cash flows are proportional to asset-1 cash flows based on $\alpha \leq 1$, in period 1 the two-asset firm will realize one of four pooled cash flow combinations: $\mathrm{U{:}\,U} \sim CF(1+\alpha)(1+\sigma)$; $\mathrm{U{:}\,D} \sim CF[(1+\alpha)+\sigma(1-\alpha)]$: $\mathrm{D{:}\,U} \sim CF[(1+\alpha)-\sigma(1-\alpha)]$: $\mathrm{D{:}\,D} \sim CF(1+\alpha)(1-\sigma)$. In period 2, as a result of the three possible payoffs to the individual assets, there are nine possible payoffs across the firm, and therefore nine possible firm value realizations based on 16 different U versus D state outcome combinations. Aggregate asset values and the various state outcome combinations are shown in columns 1-3 of Table 1.

TABLE 1 HERE

In a firm with two assets and debt that is in place, where the in-place debt was subject to the underwriting-participation constraints LPC1 and LPC2, the new lender will be required to impose its own set of underwriting-participation constraints. To be consistent with the initial constraints, the following underwriting standards are imposed at the aggregate level:

$$\underline{\text{LPC3}}: \; [\iota_1 L_1 + L_1] + [\iota_2 L_2 + L_2] \leq \frac{CF(1+\alpha)(1+\sigma)(1-\sigma)}{\delta} \tag{5a}$$

$$\underline{\text{LPC4}}: \; \iota_1 L_1 + \iota_2 L_2 \leq CF(1+\alpha)(1+\sigma) \tag{5b}$$

These underwriting-participation constraints are individually and collectively consistent with the original LPC1 and LPC2. In the cases of secured debt issuance, the constraints LPC3 and LPC4 imply that

$$\iota_2 L_2 + L_2 \leq \frac{\alpha CF(1+\sigma)(1-\sigma)}{\delta} \tag{6a}$$

$$\iota_2 L_2 \leq \alpha CF(1+\sigma) \tag{6b}$$

will be explicitly applied by the new lender. In the cases of unsecured debt issuance, collective standards LPC3 and LPC4 are applied, where the individual underwriting standards stated in (6a) and (6b) also apply, but won't necessarily bind.

With this, the following lemma identifies potential term (*t*=2) default scenarios when firm-wide asset values are referenced in making term debt payoff decisions.

<u>*Lemma 4*</u>*: Combined asset values as a function of i) U:D|D:D, ii) D:U|D:D, iii) D:D|U:D, iv) D:D|D:U, v) D:D|D:D t=2 state realizations will always be less than* $\frac{CF(1+\alpha)(1+\sigma)(1-\sigma)}{\delta}$ *, implying the possibility of term default. In the case of, vi) D:U|D:U, combined asset values will be less than* $\frac{CF(1+\alpha)(1+\sigma)(1-\sigma)}{\delta}$ *when* $\alpha < \frac{1-\sigma}{1+\sigma}$*; otherwise, combined asset value equals or exceeds* $\frac{CF(1+\alpha)(1+\sigma)(1-\sigma)}{\delta}$*. Consequently, in that case when* $\alpha < \frac{1-\sigma}{1+\sigma}$*, term default is possible. As stated in column 3 of Table 1, the probabilities associated with each of these six state outcomes are ab, ab, ab, ab,* a^2 *and* b^2*, respectively, where a and b are to be determined. Given* $\delta \geq \frac{\sigma}{4}$*, implying that cash flow at t=1 is, both individually and collectively, always sufficient to fund the debt coupon payment, continuation at t=1 is always incentive compatible regardless of the form of the debt.*

This lemma along with Table 1 will serve as a reference for analysis conducted in section V. Before moving onto that analysis, we have one final housekeeping task, which is to derive probabilities associated with the combined U versus D periodic state outcomes (the *a* and *b* quantities stated in column 4 of Table 1 and lemma 4) as they depend on the correlation between cash flow state outcomes in periods 1 and 2. To do this we identify the relevant probabilities in general terms as a function of the risk-free rate, $r \geq 0$, the cash flow shock magnitudes, σ_1 and σ_2, $\sigma_1 \neq \sigma_2$, and the correlation parameter, ρ. With the general formulations in hand, we then reimpose the restrictions $r=0$ and $\sigma_1=\sigma_2$.

The possibility of imperfect correlation follows from differences in the assets' fundamental cash flow production characteristics. Take for example a firm that invests in income-producing commercial real estate. This firm may invest in different types of commercial property—office versus hotel, for example. These two property types differ in their lease maturity structures (5-10 year leases with office versus daily leasing with hotel) and their fundamental demand determinants (white collar job growth and per worker space utilization versus leisure and business travel). Even when both assets are the same property type, cash flows may be imperfectly correlated due to, among other factors, differing lease payment adjustment structures, tenant credit quality, building location (downtown versus suburb, different city or region), building age and quality, and vulnerability to new supply.

Now consider the bivariate binomial probability distribution function as it is affected by cash flow correlation. In a given period and given two assets, let $\{p_1,p_2,p_3,p_4\}$ denote probabilities associated with the respective state realizations {U:U,U:D,D:U,D:D} in any given period. We immediately observe that $p_1+p_2+p_3+p_4=1$. Furthermore, $p_1+p_2=Pr_1[U]$ and $p_3+p_4=Pr_1[D]$ for asset-1, and $p_1+p_3=Pr_2[U]$ and $p_2+p_4=Pr_2[D]$ for asset-2. For given r, σ_1 and σ_2, by applying standard discrete-time, risk neutralized binomial pricing methods it follows that:

$$p_1 + p_2 = \frac{1}{2} + \frac{r}{2\sigma_1};\ p_3 + p_4 = \frac{1}{2} - \frac{r}{2\sigma_1};\ p_1 + p_3 = \frac{1}{2} + \frac{r}{2\sigma_2};\ p_2 + p_4 = \frac{1}{2} - \frac{r}{2\sigma_2} \quad (7)$$

Note $r=0$ recovers the risk neutral probabilities used in the single asset case.

With these relations in hand, we can derive and then use expected returns to investment fors assets 1 and 2, as well as the variances and covariance of returns, to generate three identifying structural relations for state outcome probabilities. Results are stated in the following lemma.

Lemma 5: 1) $p_1 + p_2 - p_3 - p_4 = \frac{r}{\sigma_1}$; 2) $p_1 - p_2 + p_3 - p_4 = \frac{r}{\sigma_2}$; 3) $p_1 - p_2 - p_3 + p_4 = \frac{r^2+\rho\sqrt{(\sigma_1+r)(\sigma_1-r)}\sqrt{(\sigma_2+r)(\sigma_2-r)}}{\sigma_1\sigma_2}$

Lastly, we take the three structural relations stated in lemma 5 together with the fact that $p_1+p_2+p_3+p_4=1$ to obtain the desired probabilities. Proposition 2 states the results.

Proposition 2:

$$Pr[U{:}U] = p_1 = \frac{1}{4}\left[1 + \frac{r}{\sigma_1} + \frac{r}{\sigma_2} + \frac{r^2+\rho\sqrt{(\sigma_1+r)(\sigma_1-r)}\sqrt{(\sigma_2+r)(\sigma_2-r)}}{\sigma_1\sigma_2}\right];$$

$$Pr[U{:}D] = p_2 = \frac{1}{4}\left[1 + \frac{r}{\sigma_1} - \frac{r}{\sigma_2} - \frac{r^2+\rho\sqrt{(\sigma_1+r)(\sigma_1-r)}\sqrt{(\sigma_2+r)(\sigma_2-r)}}{\sigma_1\sigma_2}\right];$$

$$Pr[D{:}U] = p_3 = \frac{1}{4}\left[1 - \frac{r}{\sigma_1} + \frac{r}{\sigma_2} - \frac{r^2+\rho\sqrt{(\sigma_1+r)(\sigma_1-r)}\sqrt{(\sigma_2+r)(\sigma_2-r)}}{\sigma_1\sigma_2}\right];$$

$$Pr[D{:}D] = p_4 = \frac{1}{4}\left[1 - \frac{r}{\sigma_1} - \frac{r}{\sigma_2} + \frac{r^2+\rho\sqrt{(\sigma_1+r)(\sigma_1-r)}\sqrt{(\sigma_2+r)(\sigma_2-r)}}{\sigma_1\sigma_2}\right].$$

When r=0, $p_1 = p_4 = \frac{1+\rho}{4}$; $p_2 = p_3 = \frac{1-\rho}{4}$.

Going forward, we assume that r=0 and $\sigma_1=\sigma_2$. These resulting simplified relations now provide definitions of previously stated probabilities a and b (see column 3 of Table 1 and lemma 4), where

$$a \equiv p_1 = p_4 = \frac{1+\rho}{4} \tag{8a}$$

$$b \equiv p_2 = p_3 = \frac{1-\rho}{4} \tag{8b}$$

V. Underinvestment and Contracting Externalities Under Alternative Debt Funding Configurations

In this section we apply results from the previous three sections to analyze implications associated with the four debt issuance alternatives available to help fund the new investment opportunity: Non-recourse, bankruptcy remote (NR-BR) secured debt; Standard non-recourse (S-NR) secured debt; Junior unsecured (J-UN) debt; and *Pari-passu* unsecured (PP-UN) debt. Our focus is primarily on wealth transfer and underinvestment that may result from debt contracting externalities. When debt contracting externalities are shut down, analyses can begin and possibly end with independent assessments of distinct assets and the associated debt. There are, however, instances of debt contracting externalities affecting secured debt, and especially S-NR debt, that require analysis across the entire firm. In the cases of unsecured debt issuance, contracting externalities are more obvious and directly depend on the scale of new investment and correlation structure of cash flows associated with assets 1 and 2. The four debt issuance alternatives are now considered in turn, and roughly in reverse order of the degree to which there are debt contracting externalities.

V.A. NR-BR Secured Debt Issuance

As shown in lemma 2, the issuance of NR-BR debt to fund new investment creates a strong two-way barrier between the two assets and the debt used to finance them. As long as $\delta \geq \frac{\sigma(1-\sigma)}{4+6\sigma}$, which includes regimes 1, 2 and part of 3 as detailed in Proposition 1, due to the incentive compatibility of default the firm has no incentive to cross-subsidize debt repayment. This implies that the issuance of new NR-BR debt can be analyzed on a standalone basis, where, as shown in lemma 2, there is no wealth transfer to the in-place debtholder. The issuance of NR-BR debt therefore fully resolves the underinvestment problem in these regions.

There is one subtle complication that was previously highlighted in Proposition 1 and the follow-up corollary. When $\delta < \frac{\sigma(1-\sigma)}{4+6\sigma}$ (the high asset value case), the cash flow underwriting-participation constraint LPC2 binds, implying that liquidity default occurs at the D node at *t*=1. But liquidity default in this region is no longer incentive compatible, implying the firm would like to fund the coupon payment gap if possible. In corollary 1, which considers only the single asset case, funding must come from outside the firm. In contrast, now that we consider the multi-asset firm, we can analyze the possibility of funding coming from within the firm.

Now, without any debt covenants to the contrary, the injection of discretionary equity funding may occur from one part of the firm to the other. But, because the lender's underwriting-participation constraint LPC2 is binding, according to the model, at *t*=1 in no state of the world for either asset will there be excess cash flow available to funnel into another part of the firm. There is, however, one other possible channel that exists to fill a coupon payment funding gap at *t*=1. The following proposition addresses that case.

<u>Proposition 3</u>*: Assume UN debt in place to finance asset-1 and NR-BR debt is chosen to finance asset-2, where the UN debt results in L_3 and $\iota_3 L_3$ as per Proposition 1. According to lemma 2, the issuance of NR-BR debt transforms the UN into NR-BR debt, where now the only difference is the debt sources is the asset scale, α. For both forms of debt, assume the presence of a call provision that allows the debt to be prepaid at par at t=1. Also assume* $\delta < \frac{\sigma(1-\sigma)}{4+6\sigma}$*, so that continuation at the D node at t=1 is incentive compatible for both sources of debt. In the case of a D:U realization at t=1, when new investment scale is such that* $\frac{3\delta(1+\sigma)}{2\sigma} < \alpha \leq 1$*, the firm will have an incentive to call the debt and sell asset-2, using the net proceeds to cover the coupon payment funding gap on the UN debt. Properly anticipated, this incentive causes the NR-BR lender to restructure the loan to restrict or eliminate the call provision, resulting in a loan structure as stated in Proposition 1. Alternatively if the call provision remains intact, in anticipation of a possible D:U state outcome and resulting call exercise, the NR-BR lender modifies the loan terms by reducing loan issuance proceeds and increasing the credit spread. The modified issuance proceeds are* $L_3^{@} = \frac{\alpha CF(1-\sigma)}{\delta}\left[1 + \frac{2\delta(1+a(1+\sigma))}{1-\sigma}\right] \leq \alpha L_3$*. When* $0 < \alpha \leq \frac{3\delta(1+\sigma)}{2\sigma}$*, there are insufficient net proceeds resulting from call exercise and asset sale to cover the coupon payment funding gap, resulting in NR-BR debt issuance proceeds of* αL_3 *as per Proposition 1. In the case of a U:D realization at t=1, there will always be a similar incentive to call the UN debt and sell asset-1 to cover the coupon payment funding gap. The inclusion of a debt service coverage default provision in both loan contracts also defeats equityholders' incentives to attempt to bridge the coupon payment funding gap.*

In the discussion that follows, we will focus on the case in which there is a D:U state realization at *t*=1. Given the following set of necessary conditions: i) high-value assets; ii) no debt service coverage default provision included with the in-place UN debt; iii) loan terms as per Proposition 1; iv) a call provision included on the NR-BR secured debt; and v) asset-2 of sufficient scale, we demonstrate an *internal* liquidity funding channel capable of compromizing the stand-alone structure of the NR-BR debt. The cross-firm contracting externality appears because there is call option value associated with cashing in positive equity value from asset-2 and redeploying that liquidity to ensure continued ownership and control of the financially distressed asset-1. When the call incentive is properly recognized by the NR-BR lender, loan proceeds decline and debt funding costs increase, to the disadvantage of the financially constrained firm. The magnitude of the loan proceeds reduction depends on $a = \frac{1+\rho}{4}$, where loan proceeds are increasing in the correlation between asset cash flow states. As $\rho \rightarrow 1$, the likelihood of a U:D outcome becomes increasingly remote, which reduces NR-BR lender call option value concerns. Alternatively, as ρ becomes smaller or possibly negative, the likelihood of a U:D realization increases to reduce loan proceeds and increase the credit spread.

As we previously highlighted in corollary 1, the UN lender will also be adverse to the firm's incentive to cover the debt funding gap, since the debt is actually more valuable given a default outcome at t=1. If the UN lender were to anticipate this possibility at the time of initial contracting, it would change the loan contract terms as per corollary 1. Alternatively, the UN lender could simply include a debt service coverage default provision that enforces liquidation given a D outcome at t=1. This by itself would defeat the cross-subsidy incentive. The elimination of the call provision or imposition of a sufficiently large prepayment penalty will also mitigate this subtle contracting externality that exists in the NR-BR debt structure.

If debt providers properly anticipate the cross-subsidy incentives of the firm, the financially constrained firm may, as a result, willingly commit to these restrictive loan provisions in order to obtain better execution at the time of loan funding. Thus our model establishes a rationale for the existence of the aforementioned debt service coverage default and call restriction provisions. In particular, our model predicts these provisions are more likely when: i) higher-valued assets are debt financed, ii) asset cash flows and values in the multi-asset firm are imperfectly correlated; and iii) asset investment scales in the multi-asset firm are comparable.

Finally, we note that this contracting externality addresses wealth transfer incentives, but is not an underinvestment problem *per se*. This is because the in-place equityholder is not harmed by the cross-subsidy incentive; rather, the in-place equityholder's wealth can actually increase from funding the coupon payment gap as initiated by early repayment on the newly issued debt.

V.B. Standard Non-Recourse (S-NR) Secured Debt Issuance

Standard non-recourse secured debt is distinguished from non-recourse bankruptcy remote debt by the fact that, in the case of S-NR debt, equity value associated with the new investment can be extracted by in-place debtholders, implying wealth transfer and potential underinvestment. To preserve space in this section while also highlighting the salient issues, we focus on model implications deriving from regimes 1 and 2 as outlined in Proposition 1.

V.B.1. Regime 1: $\boldsymbol{\delta \geq \frac{\sigma}{4}}$ (Low asset value)

In this regime there is always sufficient cash flow to fund debt coupon payment at t=1, implying that only term default can occur. Given that the two debt sources are quasi-isolated from one another, default outcomes result at the D|D node at t=2 for both assets. If default occurs with asset-2, there is nothing the S-NR lender can do to extract any excess equity value that might exist in the firm, since the debt is non-recourse. Moreover, a secured position on asset-2 implies the S-NR lender calculates loan proceeds and the credit spread according to Proposition 1, implying that debt value associated with the new investment is unaffected by the existence of payoff externalities.

However, given a D|D realization with asset-1, the UN debtholder will look to extract equity value that may exist with asset-2. In this case, a D|D state outcome with asset-2 yields no excess equity value. Neither does a U|D^D|U outcome, since full loan repayment at t=2 requires monetizing the entire cum-dividend asset value. A U|U outcome for asset-2 does, however, yield excess equity value that is accessible to the UN debtholder, occurring with probability b^2 when assessed at the time of new S-NR debt issuance (t=0). This outcome introduces the potential for a wealth transfer going from the equityholders to the UN debtholder, implying that underinvestment incentives exist. The following proposition quantifies the incentive problem.

Proposition 4: *Consider a firm with in-place UN debt, and that is contemplating the issuance of S-NR secured debt to finance the new investment opportunity. Given a D:U|D:U realization at t=2, the UN lender will look to extract residual equity value associated with asset-2. As of t=0, this state outcome occurs with probability* b^2. *The expected value of the wealth transfer from equityholders to the in-place debtholder from financing the new investment with S-NR debt is* $b^2 Min\left\{\frac{2\alpha CF\sigma(1+\sigma)}{\delta}, \frac{2CF\sigma(1-\sigma)}{\delta}\right\}$ *for 0<α≤1. Break-even occurs at* $\alpha = \frac{1-\sigma}{1+\sigma}$. *The wealth transfer is decreasing in ρ, non-decreasing in α, decreasing in δ, and increasing in σ when σ<.5.*
.

Proposition 4 quantifies the wealth transfer from equity to the in-place UN debt given the use of S-NR to finance the new investment. As noted earlier, the good news is that issuance proceeds and credit spread associated with the new S-NR debt remain unaffected. Recalling that $b = \frac{1-\rho}{4}$ as stated in equation (8b), the bad news is that equity is increasingly devalued when cash flows from assets 1 and 2 are increasingly less correlated. This generates the empirical prediction that S-NR debt is utilized more with highly correlated assets as compared to NR-BR, the latter of which shuts down the wealth transfer regardless of ρ.

The magnitude of the underinvestment problem is also affected by the scale of the new investment. When $\alpha < \frac{1-\sigma}{1+\sigma}$, there is insufficient residual equity value associated with asset-2 to fully fund the terminal UN debt payment. In this case, the firm is entirely liquidated to repay UN debtholders. However, when $\alpha \geq \frac{1-\sigma}{1+\sigma}$, there is sufficient residual equity value to fund the terminal UN debt payoff. As a result, the firm remains a going concern and retains control of both assets, with an aggregate residual equity value of $\frac{2\alpha CF\sigma(1+\sigma)}{\delta} - \frac{2CF\sigma(1-\sigma)}{\delta} = \frac{2CF\sigma[\alpha(1+\sigma)-(1-\sigma)]}{\delta}$. These outcomes are in contrast to the case of financing investment with NR-BR secured debt, where the poorly performing asset-1 is surrendered but control over well performing asset-2 is retained.

V.B.2. Regime 2: $\frac{\sigma(1-\sigma)}{4+2\sigma} \leq \delta < \frac{\sigma}{4}$ (Moderate asset value)

In this case, according to Proposition 1, there is a liquidity default for both assets given a D realization at *t*=1. Liquidity default in this case is also incentive compatible from the firm's perspective, implying no incentive to cross-subsidize coupon debt repayment. The interesting situation here happens when D:U is realized at *t*=1. In this case the firm would prefer to simply default and walk away from the poorly performing asset-1, whereas it will want to continue to own and control asset-2, which is performing well. The S-NR debt structure is, however, vulnerable to a wealth transfer from equityholders to UN debtholders, and therefore underinvestment incentives, since UN debtholders will look to available cash and possibly asset equity value in attempt to shore up cash flow and asset value deficiencies associated with asset-1.

Financial distress associated with asset-1 consequently generates an externality that disrupts the integrity of the S-NR debt structure, the intent of which is to avoid default or early prepayment given a U realization at *t*=1. Properly anticipated by the S-NR lender, this contracting externality creates an unstable situation that undermines the original debt structure, resulting in lower issuance proceeds and higher debt costs for the firm. There are also endogenous changes to the loan underwriting regime parameter boundaries, which further complicates credit risk analysis and capital structuring, implying that unsecured and standard non-recourse secured debt are not terribly compatible.

To elaborate on these issues, consider the financing of new investment with S-NR debt at t=0 according to terms set out in Proposition 1. Next consider the possibility of a D:U outcome at t=1. This results in a cash flow default on the UN debt as funded by asset-1. The shortfall causes the UN debtholder to look over to asset-2 and inquire whether sufficient excess cash flow exists to bridge the coupon payment funding gap. The following lemma answers that question.

<u>Lemma 6</u>: *Given a D:U realization at t=1, for any $0<\alpha\leq 1$ there is never sufficient excess cash flow from asset-2 after funding the S-NR coupon payment to bridge the UN coupon payment funding gap.*

.

This relation holds for asset-2 of any feasible size, $0<\alpha\leq 1$. As a result, based on a bankruptcy filing if necessary, the UN debtholder can be expected to force the liquidation of asset-2 at t=1 in order to increase its recoveries. The possibility of a forced liquidation of asset-2 at t=1 creates a payoff-contracting externality that disrupts the S-NR debt structure, which originally anticipated the loan going to term given a U outcome at t=1.

In anticipation of this potential outcome, the S-NR lender will modify the initial debt structure. A D-state outcome at t=1 for asset-2, as well as a U:U realization at t=1 results in the S-NR debtholder receiving payoffs that are unaffected by what is going on in other parts of the firm. A D:U outcome at t=1 does, however, introduce a contracting externality, where default on the UN debt causes a forced liquidation of asset-2. Proceeds from the liquidation are first allocated to the S-NR debtholder, which is paid in full based on its secured position, with excess proceeds allocated to the UN debtholder until it is paid in full. Then, if any excess proceeds remain, they go to the equityholder. In any case, the potential for a D:U outcome at t=1 creates a wealth transfer going from equityholders to the UN debtholder.

The following proposition summarizes the modified S-NR loan structure that is necessary to address spillover effects coming from other parts of the firm.

<u>Proposition 5</u>: *Given the contracting externality described above, the S-NR lender modifies the debt structure such that* $\alpha L_2^{\#} = \frac{\alpha CF(1-\sigma)}{2\delta(1+a)}[1+\delta+(1+2a)(1+\sigma)] \leq \alpha L_2$ *and* $\alpha \iota_2^{\#} L_2^{\#} = \frac{\alpha CF(1-\sigma)(\sigma-\delta)}{2\delta(1+a)} \geq \alpha \iota_2 L_2$. *Loan issuance proceeds are increasing in ρ and the credit spread decreases in ρ. This modified loan structure changes the regime boundary, where now the lower bound of regime 2 increases from* $\frac{\sigma(1-\sigma)}{4+2\sigma}$ *to* $Min\left\{\frac{\sigma(1-\sigma)}{3+\sigma+2a(1+\sigma)}, \frac{\sigma}{4}\right\}$.

As the correlation between asset cash flows decreases, the likelihood of a D:U realization at *t*=1 increases to decrease debt issuance proceeds and increase the credit spread. The reason for this is that a D:U outcome shortens the expected duration of the S-NR loan when the underlying asset is performing well—in effect, forcing a call on the debt at par in order to subsidize other parts of the firm, when the debt is currently valued at a premium to par. Thus, not only does the UN debtholder extract value from equity given a D:U outcome at *t*=1, but equityholders must contribute more equity and face increased debt costs from S-NR debtholders due to the modified loan structure.

Interestingly, as a result of this contracting externality, the lower bound of regime 2 endogenously increases from $\frac{\sigma(1-\sigma)}{4+2\sigma}$ to $\frac{\sigma(1-\sigma)}{3+\sigma+2\alpha(1+\sigma)}$. In fact, for $\sigma \geq \frac{1-2\alpha}{5+2\alpha}$, regime 2 disappears entirely, since $\frac{\sigma(1-\sigma)}{3+\sigma+2\alpha(1+\sigma)} \geq \frac{\sigma}{4}$, where $\frac{\sigma}{4}$ is the upper bound of regime 2. This occurs because, for $\delta \geq \frac{\sigma}{4}$, there is no possibility of cash flow default with the UN debt, which mitigates the need to modify the debt structure to begin with. Alternatively, given that $\frac{\sigma(1-\sigma)}{3+\sigma+2\alpha(1+\sigma)} \geq \frac{\sigma}{4}$ and $\delta < \frac{\sigma}{4}$, the S-NR lenders finds itself in regime 3 where LPC2 is binding and there is no excess cash flow at the U node at *t*=1.

Finally, now suppose that $\frac{\sigma(1-\sigma)}{3+\sigma+2\alpha(1+\sigma)} < \frac{\sigma}{4}$ so that regime 2 exists, further assuming that $\frac{\sigma(1-\sigma)}{3+\sigma+2\alpha(1+\sigma)} \leq \delta < \frac{\sigma}{4}$ so that the S-NR lender finds itself in regime 2. Given the modified debt structure identified in Proposition 5, the last issue we address in this section is whether there are enough net proceeds resulting from the liquidation of asset-2 to pay off the UN debtholder, with the firm retaining control over asset-1. The following corollary answers that question.

<u>Corollary to Proposition 5</u>: *Suppose that parameter values are such that regime 2 exists and the S-NR lender finds itself in regime 2. Given the resulting modified debt structure identified in Proposition 5, net proceeds from the liquidation of asset-2 given a D:U outcome at t=1 are sufficient to fully repay the debt if* $\alpha \geq \frac{(1-\sigma)(\sigma-\delta)}{(1+\sigma)(\sigma+\delta)}$. *This outcome allows the firm to retain control of asset-1; otherwise, the firm is liquidated and the UN debtholder incurs default losses.*

Thus, in summary, given a D:U realization at *t*=1, a wealth transfer occurs, going from equityholders to the UN debtholder. In anticipation of this outcome, the new investment opportunity may be lost if the NPV increment to new investment is sufficiently small. Loan terms associated with the new debt financing, should it occur, deteriorate relative to the case in which no contracting externalities exist. The disruption associated with asset-2 in this case, which is the better performing of the two assets, stands in contrast to NR-BR debt financing, where asset-2 continues to be controlled by the firm and the poorly performing asset-1 is stripped out and handed over to the UN debtholder. We further note that the inclusion of a debt service coverage or a call provision as analyzed in section V.A (with high asset value and the use of NR-BR debt) does not help in this situation, since continuation in the case of a D outcome for asset-1 at t=1 is not incentive compatible from the firm's perspective.

The complexities we identify in this section that are associated with combining UN debt and S-NR secured debt in a capital structure suggest that these types of debt are largely incompatible. This may explain why the use of S-NR debt is almost non-existent with most industrial firms (which commonly issue UN debt), and why publicly traded real estate firms that issue both unsecured and secured debt tend to prefer NR-BR secured debt (often through the CMBS loan market) over S-NR debt. Our results also explain why other types of firms such as private CRE enterprises, which commonly issue S-NR debt, do so almost exclusively, with capital structures that are consequently equivalent to issuing NR-BR debt (recall lemma 2).

V.C. Junior Unsecured (J-UN) Debt Issuance

To motivate this case, assume a provision exists stating that any additional debt issued by the firm will be junior to the in-place UN debt. It is well known in the literature that the issuance of such debt leads to significant underinvestment problems, due to the fact that the in-place UN debtholder has priority on cash flows and values not only on the in-place assets but also on the newly acquired asset. While this issue is well understood conceptually, there has been little work done examining effects as they depend on the scale of the new investment and the correlation structure of asset cash flows.

In this section we illustrate the salient issues by examining the case of $\delta \geq \frac{\sigma}{4}$, in which only term default occurs when there is a single asset and only one source of debt financing in place. We assume that in-place debt structure conforms to that stated in Proposition 1, as applied to regime 1. There are several sub-cases that require analysis, primarily as they depend on the scale of new investment. It also turns out that cash flow correlation structure plays a leading role when the scale of new investment is sufficiently large relative to the in-place asset.

V.C.1 Case 1: Smaller Scale Investment ($\alpha \leq \frac{2\sigma}{1+\sigma}$)

The scale of new investment is central to the current analysis because it offers a direct additional source of security for the in-place UN debt. When the scale of new investment is small relative to the assets in place, any financial distress that is associated with asset-1 will overwhelm asset-2 to leave little or nothing for the J-UN debtholder. In fact, for $\alpha \leq \frac{2\sigma}{1+\sigma}$, the J-UN lender is unwilling to make a loan at all because the credit risk is so high that it cannot satisfy the underwriting-participation constraints stated in equations (5a) and (6a).

To understand why, first consider the potential for term default associated with a poorly performing asset-1. A D|D outcome at *t*=2 for that asset results in a debt repayment shortfall of:

$$\frac{CF(1+\sigma)(1-\sigma)}{\delta} - \frac{CF\sigma(1-\sigma)^2}{\delta} = \frac{2CF\sigma(1-\sigma)}{\delta} \quad (9)$$

If this state is realized, the senior UN debtholder will look to asset-2 to help cover the repayment shortfall. If the shortfall stated in equation (9) exceeds the value to the new investment given a U|D^D|U realization at *t*=2, the senior UN will take the entire asset, leaving nothing to repay debt outstanding of any size. Thus, for any positive loan amount, the J-UN lender cannot satisfy LPC3 stated in equation (5a), and therefore withdraws. More formally, this outcome occurs when

$$\frac{\alpha CF(1+\sigma)(1-\sigma)}{\delta} \leq \frac{2CF\sigma(1-\sigma)}{\delta} \quad (10)$$

implying that $\alpha \leq \frac{2\sigma}{1+\sigma} = \alpha^A$, as stated at the beginning of this sub-section.

As a result, the new investment will have to be financed with equity or an alternative form of debt. We will focus on equity finance in order to highlight the magnitude of the underinvestment problem, noting that our no-J-UN debt financing result implies that this type of debt is particularly ill-suited for larger firms that undertake incremental investment.

In analyzing the wealth transfer from equity to the in-place UN debt, there are two sub-cases to consider. We initially focus on the subcase of $\frac{\alpha CF(1+\sigma)^2}{\delta} \leq \frac{2CF\sigma(1-\sigma)}{\delta}$, which follows when the scale of new investment is particularly small relative to the in-place asset. In this sub-case there is not enough asset-2 value given a U|U state outcome at *t*=2 to bridge the debt repayment shortfall associated with asset-1. In terms of the scale parameter, this subcase implies that $0 < \alpha \leq \frac{2\sigma(1-\sigma)}{(1+\sigma)^2} = \alpha^A\left(\frac{1-\sigma}{1+\sigma}\right)$. The following lemma states the magnitude of the underinvestment problem given equity financing associated with the new investment.

<u>Lemma 7</u>: When $0 < \alpha \leq \frac{2\sigma(1-\sigma)}{(1+\sigma)^2}$ and given parameters associated with this subcase, if equity is used to finance the new investment the wealth transfer from equity to the in-place UN debt is $\frac{\alpha CF}{4\delta}$.

.

The size of the wealth transfer is significant, proportionally equaling $\frac{1}{4(1+\delta)}$ of the new investment's value. This requires the new investment to generate value well above its cost in order to pay for the wealth transfer as well as the shadow cost of supplying the scarce equity capital for investment.

In the sub-case of $\frac{2\sigma(1-\sigma)}{(1+\sigma)^2} < \alpha \leq \alpha^A$, there is still not enough scale in the new investment for the J-UN to offer a loan, but there is now sufficient scale so that the value of asset-2 given a U|U state outcome at *t*=2 exceeds the debt repayment shortfall associated with asset-1 given a D|D outcome at *t*=2. The excess equity is internalized by the firm, thus somewhat reducing the wealth transfer to the in-place UN debtholder. The magnitude of the wealth transfer reduction is stated as follows.

<u>Lemma 8</u>: When $\alpha^A\frac{1-\sigma}{1+\sigma} < \alpha \leq \alpha^A$, and equity is used to finance new investment, the wealth transfer stated in lemma 7 is reduced by $\frac{b^2 CF[\alpha(1+\sigma)^2 - 2\sigma(1-\sigma)]}{\delta}$ as measured at the time of new investment.

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The stated reduction is seen to be decreasing in ρ, implying that, in addition to the necessary increase in scale, low-correlation asset combinations are advantageous in reducing the underinvestment problem.

V.C.2 Case 2: Larger Scale Investment ($\alpha > \frac{2\sigma}{1+\sigma}$)

New investment is now of sufficient scale that the J-UN lender is willing to participate according to LPC3. In terms of the J-UN credit analysis, there are seven *t*=2 state outcomes that potentially negatively affect the J-UN debt payoff: i) U:D|U:D; ii) U:D|D:D; iii) D:D|U:D; iv) D:D|D:D; v) D:U|D:U; vi) D:U|D:D; vii) D:D|D:U. Associated probabilities are displayed in Table 1. In order to simplify the analysis, we further restrict investment scale. The restricted range is $\frac{1-\sigma}{1+\sigma} < \alpha \leq Min\left\{\frac{2\sigma}{1-\sigma}, 1\right\}$. The lower end of this range represents an increase from $\alpha^A = \frac{2\sigma}{1+\sigma}$, and is imposed to generate a full payoff for the J-UN debtholder given a D:U|D:U state realization at *t*=2. The upper end of the range is imposed to generate a zero payoff to the J-UN debtholder given a D:D|D:D state realization at *t*=2. Given this investment scale range restriction, one additional constraint is required: $\sigma > \frac{\sqrt{20}-4}{2} = .2361$. This restriction is required to ensure the existence of the stated scale range, $\frac{1-\sigma}{1+\sigma} < \alpha \leq Min\left\{\frac{2\sigma}{1-\sigma}, 1\right\}$. Again, these additional scale restrictions are imposed to facilitate and simplify the analysis, where qualitative relations without these restrictions are highly similar.

The following proposition summarizes the main results of this subsection.

<u>Proposition 6</u>: *Given the parameter restrictions stated above, the loan amount offered by the J-UN lender is* $L^{J-UN} = \frac{CF(1-\sigma)}{2\delta}\left[\alpha(1+\sigma)\left[\frac{3}{2} - a^2 + 2b^2\right] + (\alpha - \sigma)[4ab]\right]$. L^{J-UN} *is continuously increasing in scale, α, and continuously decreasing in cash flow correlation, ρ. When ρ≥0,* $L^{J-UN} < \alpha L_1$ *as stated in Proposition 1. Alternatively, when ρ=-1,* $L^{J-UN} > \alpha L_1$*, and in fact is riskless debt with a credit spread of zero. Consequently, there exists a unique* $\rho^*, -1 < \rho^* < 0$*, such that* $L^{J-UN} = \alpha L_1$.

The proposition demonstrates that, as asset cash flows become increasingly less correlated, J-UN debt issuance proceeds increase and, when negative, eventually surpasses those associated with secured debt issuance as identified in Proposition 1. In the extreme, the debt becomes riskless with a zero credit spread, demonstrating that, rather unexpectedly, circumstances exist when J-UN debt generates superior issuance proceeds and borrower credit costs as compared to a secured debt issuance. The circumstances are admittedly uncommon. There is, however, a wealth transfer from equity to the in-place UN debtholder whenever *ρ*>-1.

The stated loan structure is potentially unstable. This happens because, as loan issuance proceeds decline when cash flow correlation increases, left unchecked the coupon payment on the J-UN loan eventually becomes large enough to cause a joint cash flow default at *t*=1. This possibility introduces a negative contracting externality flows to the in-place UN debtholder, which will cause it to include a loan provision placing an upper limit on J-UN debt issuance proceeds and coupon payments. The following corollary summarizes the result.

<u>Corollary to Proposition 6</u>: *For ρ=1, and given the loan structure identified in proposition 6 as well as a D:D outcome at t=1, there is insufficient joint cash flow to fund coupon payments for both the in-place UN debt and the J-UN debt. In particular, given the continuity of* L^{J-UN}*, there exists a* $\rho^{**}, \rho^* < \rho^{**} < 1$*, such that joint cash flow at t=1 given a D:D outcome exactly equals the joint coupon payments due on the two debt sources.*

Summarizing, our results show that J-UN debt is most problematic for smaller scale projects and for projects whose cash flows are closely correlated with cash flows from in-place assets. Underinvestment problems relax as scale increases and correlation decreases, suggesting that in extreme cases there may be a strategic role for J-UN debt. But underinvestment frictions are never completely eliminated with the issuance of J-UN debt, which combined with the identified complexity associated with structuring such debt, suggests it is generally an inferior debt form with respect to accommodating new investment.

V.D. *Pari Passu* Unsecured (PP-UN) Debt Issuance

Last but by no means least, we consider the issuance of *pari passu* unsecured debt to finance the new investment. The ability to issue PP-UN debt implies that, unlike the previous case, there is no in-place provision requiring subsequent debt issuances to assume a junior position. Rather, there is now a provision stipulating that a new debt issuance cannot assume a senior position in the capital structure. As a result, all else equal, the new lender minimizes its credit exposure by structuring a PP-UN debt issuance, which consequently maximizes issuance proceeds for the firm.

The issuance of PP-UN debt to finance the new investment implies that, once the issuance occurs, all assets and their cash flows are pooled together to support a unified single debt source. Default outcomes will therefore depend on joint payoffs to the assets relative to the joint repayment obligations. Although this suggests analysis can proceed *as if* a joint issuance is undertaken to finance the new investment, it ignores the fact that the in-place debt remains in-place, subject to the loan's original terms and conditions. Consequently, terms and conditions for the new debt issuance are determined *conditional* on the in-place debt remaining in-place and the fact that loan repayment decisions will be governed by joint asset payoff and debt repayment considerations. Contracting payoff externalities associated with the new debt issuance in relation to the in-place debt will therefore result, with a reallocation of debt and equity values.

Analysis of the new debt issuance proceeds in two steps. First, based on the underwriting-participation constraints laid out previously, aggregate debt issuance proceeds are determined *as if* this were a single new, multiple asset PP-UN debt issuance. This generates the market value for the overall debt structure. This step is essential, in that it allows us to identify joint debt repayment versus default outcomes, along with the associated payoffs as they depend on joint cash flows, asset values and stated contract repayment requirements. Determination of a joint loan market value and coupon payment amount is also useful to isolate gains or losses related to the individual debt issuances that result from contracting externalities. Then, in a second step, the new debt issuance proceeds and coupon payments are determined based on a contract that satisfies both the individual and joint lender underwriting-participation constraints as seen in equations (5) and (6), respectively.

To streamline matters while highlighting the important points, in this sub-section we analyze only the regime 1 loan structure in which $\delta \geq \frac{\sigma}{4}$. This is the case where only term default occurs. We further restrict the scale of new investment, α, so that $\frac{1-\sigma}{1+\sigma} < \alpha \leq 1$. With this new investment scale, there is sufficient collateral value to repay the UN debt conditional on a U:D|U:D realization as well as a D:U|D:U realization. New investment of smaller scale results in default given the latter outcome, which slightly complicates the analysis without providing much additional insight.

We now determine loan issuance proceeds and coupon payment *as if* the UN debt were newly issued, with both assets 1 and 2 collateralizing the loan. Given that only term default occurs in this case, our first task is to identify t=2 state outcomes that result in full versus partial debt repayment of $\frac{CF(1+\alpha)(1+\sigma)(1-\sigma)}{\delta}$. Recalling that this maximum repayment amount is set as per LPC3 stated in equation (5a), reference to Figure 3 and lemma 4 indicate there are five t=2 state outcomes that generate a default: i) U:D|D:D; ii) D:U|D:D; iii) D:D|U:D; iv) D:D|D:U; v) D:D|D:D. The associated payoffs follow from referencing Figure 3 and Table 1. State outcome probabilities are, respectively, ab, ab, ab, ab, a^2. With this information, combined loan issuance proceeds and coupon payment can be determined, with the following lemma summarizing the result.

<u>Lemma 9</u>: *Based on the parameter and underwriting-participations constraint restrictions stated previously, if the PP-UN debt were to be newly issued with the debt cross-collateralized by assets 1 and 2, loan issuance proceeds would be* $L^{PP-UN} = \frac{CF(1+\alpha)(1-\sigma)}{\delta}\left[1+\sigma\left(\frac{3}{4}+b^2\right)\right]$ *and the coupon payment would be* $\iota L^{PP-UN} = \frac{CF(1+\alpha)\sigma(1-\sigma)}{4\delta}[1-4b^2]$. *Loan amount (credit spread) is decreasing (increasing) in ρ, with* $L^{PP\text{-}UN} \geq L_1$ *and* $\iota L^{PP\text{-}UN} \leq \iota L_1$, *where* L_1 *and* ιL_1 *are determined according to Proposition 1 with the scale of the asset adjusted equal to* $1+\alpha$.

Only when there is perfect positive correlation between asset cash flows does the PP-UN loan structure replicate the single loan structure as defined in Proposition 1. Otherwise, imperfect asset cash flow correlation reduces aggregate credit risk to increase combined PP-UN debt value relative to the joint value of fully separated (NR-BR) loan structures. Joint coupon payments and credit spread are consequently lowered relative to the fully separated case. These changes follow because default no longer occurs given the state outcomes of U:D|U:D and D:U|D:U, both of which occur with probability b^2.

Given that the in-place debt remains in place, the act of financing the new investment with PP-UN debt introduces a contracting externality that affects both the in-place debt and the newly issued debt. With respect to the in-place debt, the elimination of default given a D:U|D:U state outcome reduces credit risk, as does the allocation of default losses to the new debtholder when D:U|D:D or D:D|D:U state realizations occur. On the other hand, the introduction of default and allocated losses given U:D|D:D and D:D|U:D state outcomes increases credit risk. We will show the net effect results in a wealth transfer from equity to the in-place debtholder. In terms of valuing the newly issued debt, similar tradeoffs occur, but in this case the effects are internalized so that the firm shares in the benefit of a net reduction in credit risk.

We are now in position to calculate issuance proceeds and coupon payments associated with the newly issued debt. Here, the first step is to recall the in-place loan was originally underwritten in accordance with LPC1, with $\iota L_1 + L_1 = \frac{CF(1+\sigma)(1-\sigma)}{\delta}$. This in-place loan, as well as the new loan, are governed by a distinct loan contracts, even though joint asset cash flows and values will be considered when making default versus debt repayment decisions. As a result, to maximize issuance proceeds, LPC3 binds and

$$\iota L_2^{PP-UN} + L_2^{PP-UN} = \frac{\alpha CF(1+\sigma)(1-\sigma)}{\delta} \tag{11}$$

where we note that $(\iota L_1 + L_1) + (\iota L_2^{PP-UN} + L_2^{PP-UN}) = \frac{(1+\alpha)CF(1+\sigma)(1-\sigma)}{\delta}$.

New loan issuance proceeds and coupon payment can now be calculated based on satisfying the equality stated in equation (11) together with a calculation of the new loan value conditional on its expected payoffs. Because payoffs in default are the result of allocating pooled asset values, whereas there are actually two separate loan contracts with their own debt repayment provisions, a loss recovery allocation rule needs to be specified. This type of rule must be reasonable and fair enough to survive a bankruptcy review process. Relative investment scale is the natural allocation decision rule, particularly since, conditional on default occurring at time *t*=2, according to LPC3 the in-place debtholder is promised a proportion $\frac{1}{1+\alpha}$ of the total payoff amount while the new debtholder's proportional payoff is $\frac{\alpha}{1+\alpha}$ of the total. With this allocation rule, the following proposition summarizes the salient new loan characteristics.

<u>Proposition 7</u>: *Issuance proceeds and coupon payment for the new loan are exactly proportional to the pooled values stated in lemma 9, where* $L_2^{PP-UN} = \frac{\alpha CF(1-\sigma)}{\delta}\left[1+\sigma\left(\frac{3}{4}+b^2\right)\right]$ *and* $\iota L_2^{PP-UN} = \frac{\alpha CF\sigma(1-\sigma)}{4\delta}[1-4b^2]$. *New loan issuance proceeds (coupon payments) are greater than (less than) those which obtain from a stand-alone (e.g., NR-BR) debt financing whenever* $\rho<1$*. As a result, the PP-UN debt structure is stable. However, whenever* $\rho<1$*, new loan issuance proceeds (coupon payments) are less than (great than) those which result from a simple differencing of* $L^{PP\text{-}UN}$ ($\iota L^{PP\text{-}UN}$) *as stated in lemma 9 and the in-place loan amount,* L_1*, (coupon payments,* ιL_1*).*

New loan issuance proceeds exceed stand-alone (NR-BR) debt issuance proceeds by $b^2\frac{\alpha CF\sigma(1-\sigma)}{\delta}$, which is increasing in the scale of investment as well as diversification of the collateral asset pool. The exact proportionality of new PP-UN loan issuance proceeds with the aggregate PP-UN loan value stated in lemma 9 implies that the firm does benefit from the asset pooling-diversification effect, but does not fully share in the credit risk reduction benefits that are introduced as a result on the PP-UN debt structure. That is, $L_2^{PP-UN} + L_1 < L^{PP-UN}$ when $\rho<1$.

The reason for the latter result is that, because of contracting externalities associated with the new PP-UN debt issuance, there is a wealth transfer from equity to the in-place UN debt. The exact amount of the wealth transfer is stated in the following corollary:

<u>Corollary to Proposition 7</u>: *The wealth transfer from equity to the in-place UN debtholder is exactly proportional of* L^{PP-UN}*, equaling* $b^2\frac{CF\sigma(1-\sigma)}{\delta}$*. This quantity equals or exceeds the net increase in new loan issuance proceeds relative to stand-alone issuance proceeds of* $b^2\frac{\alpha CF\sigma(1-\sigma)}{\delta}$*, where the difference between the wealth transfer and incremental new loan issuance proceeds is decreasing in the scale of new investment,* α*, as well as the asset diversification parameter,* ρ*.*

The wealth transfer to the in-place UN debtholders introduces the potential for underinvestment. Relative to issuing NR-BR debt, underinvestment incentives may be partially offset by the increased issuance proceeds, which reduces the shadow cost of supplying costly equity capital to fund the new investment. The offset is decreasing in the scale of new investment, which happens because, according to Proposition 7, new loan issuance proceeds are exactly proportional to the scale of new investment. The difference between the wealth transfer and incremental issuance proceeds is also increasing in *b*, implying that, rather unintuitively, collateral asset diversification effects decrease the benefits of issuing PP-UN debt relative to stand-alone NR-BR debt. Altogether, based on the existing model structure, it would seem that PP-UN debt issuance is inferior to the NR-BR debt structure, which does not suffer from wealth transfer problems, albeit NR-BR debt generates lower issuance proceeds than the PP-UN debt issuance alternative.

Note that the credit spread on the new PP-UN debt issuance is less than or equal to the credit spread that would be realized on a stand-alone NR-BR debt issuance. This happens because of the asset pooling diversification effects identified previously, together with the fact that in-place debt was valued at par prior to the announcement of the new debt financing. If instead the in-place debt was valued at a discount to par, the wealth transfer from equity to the in-place debtholder may be magnified to increase the credit spread on the new PP-UN debt issuance (recall the analysis of J-UN that resulted in extreme wealth transfers). As a result, our model is capable of generating the result of, as analyzed conditional on the value of the in-place debt, credit spreads on the new UN debt may exceed or fall below those that would be realized on an otherwise equivalent stand-alone NR-BR debt issuance.

Finally, we observe that, because coupon payments from the new PP-UN debt financing are never larger than the payments from a stand-alone debt issuance of scale α, the PP-UN debt structure is stable. By this we mean that the introduction of a new PP-UN debt issuance does not adversely effect the lower bound of regime 1, which separates term default outcomes (occurring only at t=2) from cash flow-liquidity default outcomes (occurring at t=1). Viewed another way, should parameters be such that cash flow default is feasible with the in-place debt, the addition of PP-UN is capable of eliminating cash flow default under certain conditions due to the fact that the upper bound of the cash flow default region decreases.

These outcomes stand in marked contrast to the previously analyzed S-NR and J-UN debt structures, which introduce instability and considerable complexity into the analysis of new debt financing. This leads us to conclude that the polar opposite debt structures of NR-BR, which creates a strong separation in the mapping from asset collateral to debt payoffs and default outcomes, and PP-UN, which creates pure pooling in the mapping from asset collateral to debt payoffs and default outcomes, are the most stable, least complex, and therefore the most accommodative debt structures to utilize when financing new investment conditional on there being in-place UN debt.

VI. The GGP Bankruptcy

In this final section we review and summarize the General Growth Properties (GGP) bankruptcy, which is then analyzed through the lens of our model. The events leading up to the Chapter 11 bankruptcy filing, along with events that occurred during the bankruptcy, illustrate many of the managerial objectives and tensions we highlight—most particularly as they apply to the use of non-recourse bankruptcy remote (NR-BR) secured debt to finance asset investment. For background regarding material facts and dates, we primarily rely on Pagliari (2011), which is an authoritative case study addressing events and issues that occurred prior to, during and after the bankruptcy filing.

VI.A. Background

GGP was (and still is) a publicly traded firm that primarily owned retail shopping malls. The quality of the malls as measured by their sales per square foot varied significantly, ranging from some of the most profitable malls in the US to lesser quality stuff that catered to lower-middle tier markets.[8] GGP's corporate form was that of a real estate investment trust (REIT), having gone public as such in 1993. For our purposes, the two most important characteristics of REITs are that they: i) don't pay taxes at the entity level as long as they, ii) distribute (effectively) all of their taxable income as dividends to shareholders.[9]

On the real operating and asset ownership side, we note there are serious questions as to whether economies of scale, or any other type of synergistic qualities, exist in the commercial property ownership business. Operating expenses are generally linear in scale, although there may be some small gains as they relate to general and administrative (G&A) expenses incurred at the firm level. On the revenue side, it is very hard to

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[8] Based on a Bloomberg article posted in February 2018, GGP owns three of the top five most valuable malls in the US: Ala Moana (#1) in Honolulu, Oakbrook (#3) in Chicago, and Fashion Show (#5) in Las Vegas.

[9] The dividend payout requirement is actually 90 percent of net income, but virtually all REITs pay out at least 100 percent of income to avoid taxes on the positive difference between 100 percent and the actual payout percentage.

own and control assets of sufficient scale in a local geographical area to effect rental prices. Mall property may be an exception, however. Well-located large retail malls operate almost as if they are an island retail economy, under the ownership of a single entity, and hence possess monopolist characteristics. Retailers will compete over occupancy and location within a well-located mall that has a proper mix of retail stores and local consumer demographics. Retail mall landlords that own a number of higher quality assets that possess distinct and desirable locational attributes will be in a position to potentially generate higher rental rate outcomes by cutting deals with retailers across mall locations, and excluding retailers who “don’t play ball.” In other words, the retail mall landlords possess pricing power with respect to setting rents with many of their retail tenants. It is likely that GGP possessed at least some of these characteristics.

GGP’s liability structure leading up to the bankruptcy was somewhat complicated, where its primary characteristics were high leverage and a heavy reliance on NR-BR secured debt financing. Leverage levels had increased in the years leading up to the 2007-08 financial crisis due to several LBO transactions, with leverage at the firm level exceeding 70 percent in 2007. This compared to the retail REIT industry average of about 50 percent. Ninety percent of GGP’s debt was secured, mostly as NR-BR debt that was packaged into CMBS. This 90 percent secured debt percentage is high compared with a 50 percent level employed by SPG—a firm which commonly issued PP-UN debt in addition to NR-BR secured debt.

GGP preferred shorter debt maturities, based on their expectation of increasing property prices and the consequent ability to extract surplus equity at the time of the next refinancing. In particular, GGP had average debt maturities of 2 to 3 years, while SPG, its main competitor and the largest retail REIT, had average debt maturities of around 7 years. GGP’s high leverage, heavy reliance on NR-BR debt, and shorter debt maturity structure began to cause problems for the firm in 2007 as property values stagnated and financial market liquidity became volatile and increasingly scarce. This caused GGP to engage in increasingly desperate measures to refinance large quantities of its maturing debt. As events moved into 2008, a dichotomy developed within the firm between the higher quality assets, which continued to perform well and were more easily refinanced, and the lower quality assets, which experienced revenue declines and became extremely difficult if not impossible to refinance.

As capital market conditions deteriorated in 2008, GGP found themselves mired in a full-blown liquidity crisis. Finally, after the Lehman failure, the CFO was fired in the fall of 2008 and the remaining top management was fired in early 2009. GGP filed for bankruptcy in April 2009. At the time of the filing Pagliari (2011) reports that GGP's operating fundamentals were strong overall, with good aggregate cash flow. Consequently, the firm as a whole appeared to be solvent. The problem was that the NR-BR debt had created a Balkanized firm, parts of which were performing quite well and other parts of which were not. Without an ability to subsidize the poorly performing parts of the firm (due to an "inflexible debt structure") during the middle of a liquidity crisis, a structure that management had quite intentionally created, GGP found itself in deep trouble.

After the bankruptcy filing, the bankruptcy judge, Judge Gropper, made a series of rulings that added flexibility into the existing structure, and in the process consolidated and recharacterized the NR-BR debt. He also enforced a comprehensive cramdown of the in-place debt, which amounted to an across-the-board forced debt restructuring.[10]

The consolidation and recharacterization of the NR-BR debt occurred in three interconnected way. First, the judge suspended and consolidated all "cash trap" and "reserve" accounts associated with the secured debt, which clearly breached the "fortress walls" set up by the SPE-BR entities, thus freeing up significant cash that would otherwise remain contained within SPE-BR structure.[11] Second, a centralized cash management account was created, which established a mechanism to aggregate any and all available cash from within the firm, to be reallocated as necessary as part of the centralized bankruptcy process.[12] Third, dividend payouts of any kind were suspended, which, while not unusual for bankrupt firms in general, represented a significant departure from the usual requirement that REITs pay out most or all of their net income as dividends.[13] This dividend suspension—with most of the retained cash coming from the well performing, higher quality assets that were

[10] Cramdown was implemented in one (or more) of three ways: i) principal reduction; ii) interest rate reduction; and most commonly iii) maturity extension, where maturity extension was the favored mechanism.
[11] According to Pagliari (2011), NR-BR debtholders objected to this, "as that would in effect force them to subsidize any under-performing assets."
[12] As noted by Pagliari (2011), "GGP was structured in such a way that surplus cash was 'up-streamed' from the project-level entities to a central cash asset which operated for the benefit of the entire enterprise."
[13] Even during the bankruptcy process, at the firm level GGP remained solvent, albeit illiquid.

financed with NR-BR secured debt—created an additional important source of liquidity for the firm during the restructuring process.

Dividend suspension as a recharacterization of NR-BR debt deserves some additional commentary.[14] Recall from the model that bankruptcy remoteness protects the equity that is specifically associated with the secured asset from involuntary transfer to other claimholders within the firm. In the case of REITs, onerous dividend payout requirements to the in-place shareholders actually act as an enforcement mechanism, since the cash cannot (mostly) remain within the firm. Instead, the cash must be (mostly) distributed to outside shareholders. This forced equity distribution therefore prevents the equity from reallocation, forcing the firm to go back to the capital markets for any additional funding. Dividend suspension breaks the enforcement mechanism, allowing asset-specific equity to be pooled and reallocated as the bankruptcy trustee sees fit.

The long and short of it was that the imposed structure worked to help resurrect GGP, which emerged from bankruptcy in late 2010. Throughout the bankruptcy, and in the months leading up to the filing, management made the case for the validity of a Chapter 11 bankruptcy process, with its automatic stay provision, a strong bias towards preserving the firm as a going concern, and a centralized command and control structure that is premised on preservation of the whole through sacrifice of the many financial stakeholders, including in this case NR-BR secured debtholders. Implicit in the Chapter 11 bankruptcy "bargain" is that there is actually going concern value—i.e., real economic value that would otherwise be destroyed with the dismantling of the firm. Judge Gropper, as revealed by his decisions and actions, was certainly sympathetic to this view. He was also undoubtedly influenced by an almost unprecedented ongoing liquidity crisis, with real estate assets and debt apparently at the center of it.

VI.B. Analysis

In combination, the Judge's three methods noted above—suspension and consolidation of the cash trap and reserve accounts, creation of a centralized cash management account, and suspension of the otherwise required dividend payout requirement—stripped, at least temporarily, the NR-BR secured debt of its bankruptcy

[14] This aspect of the GGP bankruptcy was not emphasized in Pagliari (2011).

remoteness. What it left was debt that more closely resembled standard non-recourse debt. As analyzed in section V.B., S-NR secured debt is vulnerable to equity value stripping when outsider claimholders have a financial incentive and legal standing to invade the structure. While GGP's secured debtholders would have seemed to remain well protected, particularly since the (often higher quality) assets stayed in place as security, if such an outcome is anticipated *ex ante* the integrity of the NR-BR debt structure is compromised due to the introduction of contracting externalities.

Indeed, as pointed out by Baird and Casey (2013) and others, the SPE-BR structure is a mechanism to "opt out" of the normal bankruptcy process. We show in our analysis of *pari-passu* unsecured debt that, due to underinvestment incentives that increase in the degree of asset diversification, the NR-BR debt structure is most suitable for firms that hold a diversified set of assets. This certainly seems to be the case with GGP, which held a wide variety retail assets based on quality, location, age and size characteristics. The source of GGP's financial management mistakes consequently seemed less about the *type* of debt utilized than about the *level* of financial leverage.[15] Indeed, GGP had the highest leverage of any retail REIT going into the financial crisis. At the same time many of its direct competitors, including SPG, made liberal use of NR-BR debt. Further, as shown in our analysis of S-NR debt in section V.B., wealth transfer and structural complexity *increase* as asset diversification increases, to undermine the structure.

A more subtle *ex ante* effect was considered in section III. When considering debt that is secured by a high-value (low δ) asset, we show that default is no longer incentive compatible from the firm's perspective given that illiquidity was the cause of the default. This implies the firm will have incentives to use other sources of liquidity to bridge the debt payment funding gap in order to retain control of the asset. If the firm can credibly commit *not* to fund debt repayment conditional on a liquidity default, which we show is possible by incorporating debt service coverage default and restrictive call provisions into the secured debt contract, the borrower can expect an increase in loan proceeds and a reduced credit spread on the debt. This description seems to fit GGP's circumstances well, where the recharacterization of secured NR-BR debt (and repudiation of

[15] As pointed out by Riddiough and Steiner (2017), non-recourse secured debt lacks a commitment mechanism for maintaining lower leverage levels across the firm. This type of mechanism does exist with unsecured debt, due to the inclusion of provisions limiting total debt and the use of secured debt. With their almost exclusive use of secured debt, GGP seems to have succumbed to the charms of high leverage, as also pointed out by Riddiough and Steiner.

the call restriction andn default trigger provisions) introduces an additional potential source of inefficiency when such recharacterization undermines benefits associated with the initial debt structure.[16]

We should reiterate that the dividend payout component of the REIT structure enhances the integrity of the NR-BR secured debt structure. In fact, according to our model, the dividend payouts are necessary—otherwise, the structure assumes the character of S-NR debt in which available equity is vulnerable to extraction by outside creditors. This may explain why REITs (and other private, project-oriented firms that regularly pay out dividends, with little or no cash retained within the firm) use NR-BR debt, but other publicly traded firms in other industries that are can retain earnings do not. As the GGP bankruptcy aptly illustrates, if excess equity from NR-BR financed projects is retained and pooled, rather than paid out as a dividend, with that retained cash made available to subsidize financially distressed assets as necessary from other parts of the firm, the equity protection/no-wealth transfer feature of bankruptcy remoteness is defeated.

The current version of our model does not explicitly account for strategic complementarities gained from vertical integration and that may be specific to the firm and the in-place management team. The existence of such effects suggest significant positive correlation in asset cash flows, whereas unrelated products (held, say, in a conglomerate structure) and products that have unique locational and quality characteristics such as commercial real estate, exhibit fewer strategic complementarities. But these kinds of assets often generate greater diversification effects. A tradeoff between strategic complementarities and diversification effects therefore exists, reinforcing our conclusions regarding the benefits of NR-BR secured debt relative to the PP-UN debt form. That is, because the break-up costs associated with diversified assets are generally lower than those that exhibit strategic complementarities, NR-BR debt is better matched with less synergistic and more diverse assets, while PP-UN debt is better matched to firms that a hold a set of similar and complementary assets.

Traditional justifications for Chapter 11 bankruptcy rely on the firm possessing significant going concern value that would be lost if management and assets were separated through liquidation. Although the retail properties held by GGP may have exhibited some scale economies or other synergistic effects, these

[16] In addition to extending loan maturities and suspending default provisions such as debt service coverage ratio tests, Judge Gropper also relaxed call restrictions by requiring NR-BR lenders waive their usual (stiff) prepayment penalities. As part of the debtor-in-possession financing package, GGP took full advantage of their newfound call option by prepaying many of their performing NR-BR loans at par or discounts to par.

effects were not due to special talents of the assembled management team. Rather, the effects were associated with the assets themselves. While owning and managing such assets does take specialized experience, at the time of GGP's bankruptcy there was no dearth of available talent from within nor outside the retail REIT investment world. Indeed, there is little question that a firm like SPG could (and in fact wanted to) acquire the entire portfolio of GGP assets and manage them at least as efficiently as the GGP management.[17]

This leaves us with the issue of whether it would be appropriate to liquidate GGP in the middle of a liquidity crisis. Going back to at least Bagehot (1873), it is well understand and generally accepted that financial institutions should not be liquidated in the middle of financial panic, where it is difficult to distinguish between insolvency and illiquidity. A similar rationale can be asserted with non-financial firms, but with the qualifier that it is necessary for the *entire sector-industry* to simultaneously experiencing financial distress (as also happens to financial institutions in general financial panics). When this occurs, one can appeal to Shleifer and Vishny (1992) arguments regarding the inefficiency of asset firesales. The firesale argument hinges, however, on the entire industry being sidelined at the same time, with distressed assets being sold to outsiders that operate them less efficiently than insiders would if they only had the means to acquire them.

As applied to GGP and the retail property sector during the bankruptcy period that went from April 2009 to late 2010, it is true that, at least during the first half of 2009, the industry was experiencing a fair bit of distress. But, as highlighted in Packer et al (2012), what happened in the commercial property sector was far different than in housing. Commercial property REIT share prices bottomed out in March 2009 and increased sharply thereafter. Although other retail REITs such as SPG were not unaffected by the dearth of liquidity in capital markets, these firms did not experience any severe financial distress. Indeed, SPG had positioned itself to be a buyer of retail mall properties should the opportunity present itself, as did other retail firms and opportunistic investors. In fact, SPG offered to acquire GGP, but was turned away. Had a liquidation occurred

[17] Some have argued that acquisition of GGP's assets by SPG would have triggered a review by the FTC. While SPG was and still is the largest retail REIT and owner-operator in the mall business, there would have remained significant competition in the retail space (particularly when the disruptive effects of the internet are recognized). Based on details contained in Pagliari (2011), as well as attempts by SPG to acquire other retail REITs, the issue of monopolistic control looks to be a red herring, where the real issue seems to be entrenched managers looking for ammunition to defeat a possible takeover.

instead, SPG, as well as other sophisticated retail mall owner-operators, would have certainly bid on many of the assets, and in the process had sufficient liquidity to operate them efficiently.

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Figure 1

Evolution of Cash Flows Over Time

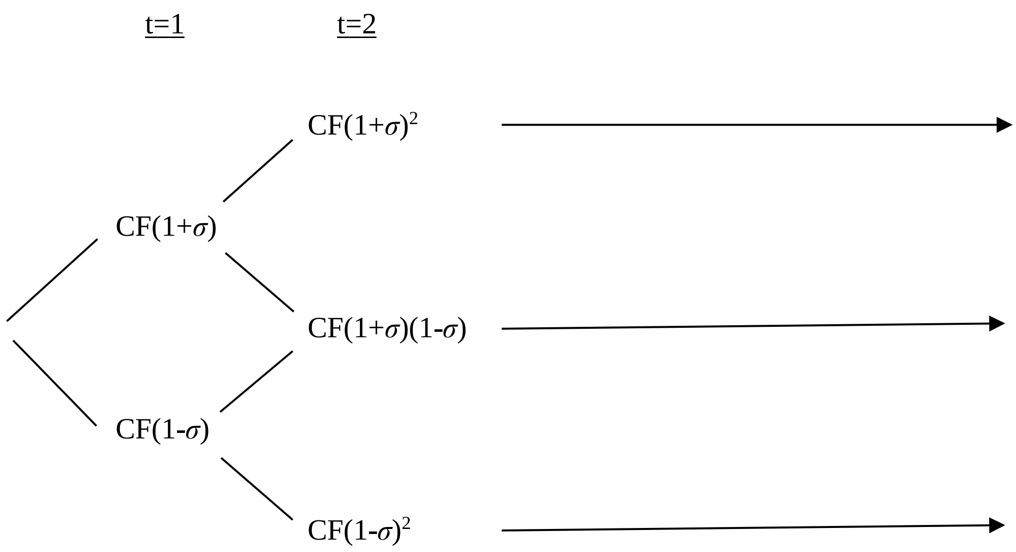

Figure 2

Evolution of Asset Values Over Time

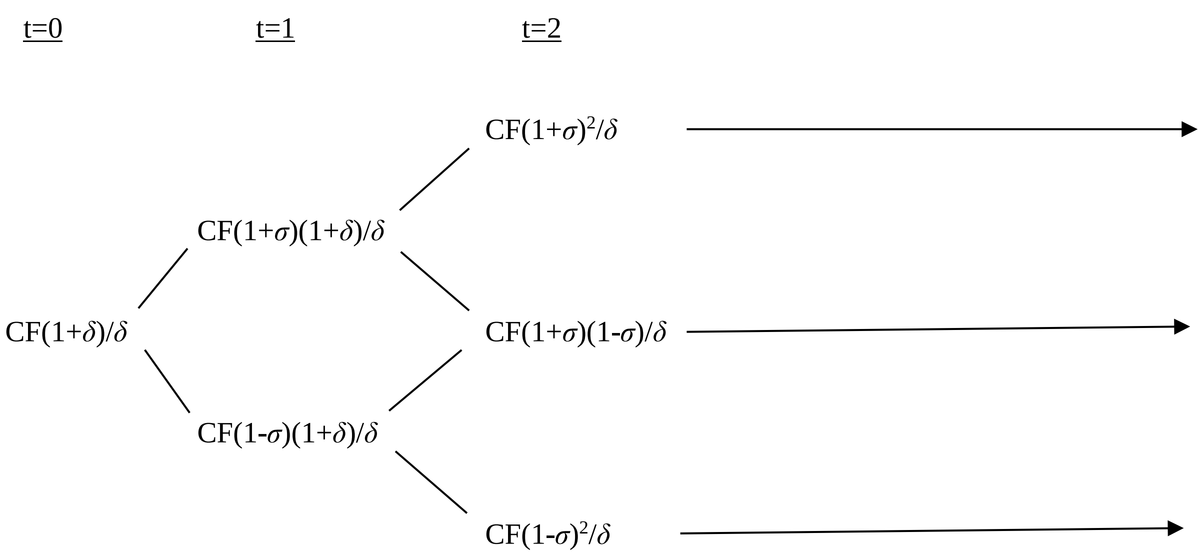

Figure 3

Regimes and Default Nodes (Circled)

Panel A: Regime 1 — $\boldsymbol{\delta \geq \sigma/4}$

Panel B: Regime 2 — $\boldsymbol{\sigma(1-\sigma)/(4+2\sigma) \leq \delta < \sigma/4}$

Panel C: Regime 3 — $\boldsymbol{\delta < \sigma(1-\sigma)/(4+2\sigma)}$

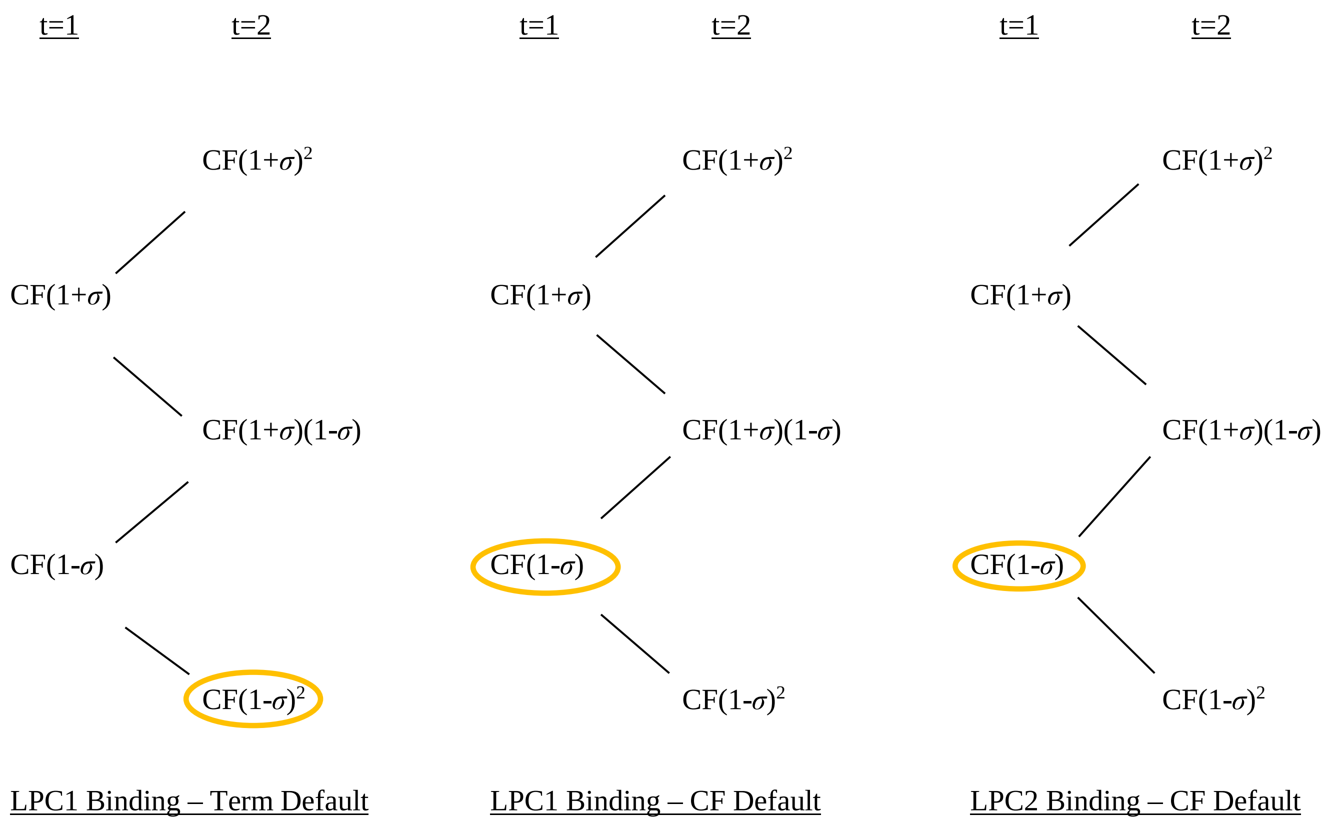

Table 1

2-Asset State Realizations, Combined Cash Flows, and Probabilities

Combined State Realization t=1 | t=2	Combined Cash Flow t=2	CF State #	Probability
U:U | U:U	$CF(1+\alpha)(1+\sigma)^2$	1	a^2
U:U | U:D	$CF(1+\sigma)^2+\alpha CF(1+\sigma)(1-\sigma)$	2	ab
U:U | D:U	$CF(1+\sigma)(1-\sigma)+\alpha CF(1+\sigma)^2$	3	ab
U:U | D:D	$CF(1+\alpha)(1+\sigma)(1-\sigma)$	4	a^2
U:D | U:U	$CF(1+\sigma)^2+\alpha CF(1+\sigma)(1-\sigma)$	2	ab
U:D | U:D	$CF(1+\sigma)^2+\alpha CF(1-\sigma)^2$	5	b^2
U:D | D:U	$CF(1+\alpha)(1+\sigma)(1-\sigma)$	4	b^2
U:D | D:D	$CF(1+\sigma)(1-\sigma)+\alpha CF(1-\sigma)^2$	6	ab
D:U | U:U	$CF(1+\sigma)(1-\sigma)+\alpha CF(1+\sigma)^2$	3	ab
D:U | U:D	$CF(1+\alpha)(1+\sigma)(1-\sigma)$	4	b^2
D:U | D:U	$CF(1-\sigma)^2+\alpha CF(1+\sigma)^2$	7	b^2
D:U | D:D	$CF(1-\sigma)^2+\alpha CF(1+\sigma)(1-\sigma)$	8	ab
D:D | U:U	$CF(1+\alpha)(1+\sigma)(1-\sigma)$	4	a^2
D:D | U:D	$CF(1+\sigma)(1-\sigma)+\alpha CF(1-\sigma)^2$	6	ab
D:D | D:U	$CF(1-\sigma)^2+\alpha CF(1+\sigma)(1-\sigma)$	8	ab
D:D | D:D	$CF(1+\alpha)(1-\sigma)^2$	9	a^2