Search Frictions, Fund Manager Skill, and Net-of-Fee Performance in Private Equity

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Abstract

This paper develops a general equilibrium model of directed search between institutional investors and private equity fund managers to explain the variation in fund size, market tightness, and net-of-fee profits across fund managers. Skilled fund managers oversee larger funds in tighter submarkets, achieving higher net-of-fee profits. Costly signaling due to asymmetric information about fund manager skill increases fund size and market tightness. The model also accounts for investor demand for smoothed interim returns, which can explain negative alphas observed in the data. Empirical analysis supports the model's novel predictions, including a positive relation between fund size (proxying for fund manager skill) and net-of-fee profits as well as market tightness.

Keywords: Search Theory, Private Equity, Pension Funds, Private Equity Real Estate, Adverse Selection, Deep Neural Networks

JEL Codes: C45, D82, D83, G11, G23, G24, G32, L26

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Institutional investment in closed-end Private Equity (PE) funds is characterized by a search and matching process that occurs between institutional investors (Limited Partners, LPs) and delegated fund managers (General Partners, GPs). Previous research on search frictions in private asset markets has primarily focused on fund managers' search for projects in venture capital (Sørensen, 2007; Silveira and Wright, 2016; Fox et al., 2018) and in commercial real estate markets (Sagi, 2021). The implications of LP-GP search and matching frictions on PE market outcomes remain unexplored, however.

This paper is motivated by, and directly extends, the seminal work of Berk and Green (BG, 2004) and Berk and van Binsbergen (BvB, 2015). BG provide the first rational model that reconciles the existence of skilled mutual fund managers with findings of zero average net-of-fee alpha. In the model, investors learn about fund manager skill and elastically supply capital to managers that outperform. Skilled managers, who, based on prior performance, are increasingly recognized as skilled, manage larger funds and charge higher fees to fully extract the rents they create. Net-of-fee alpha is therefore zero on average, and there is no persistence in fund performance based on benchmarked net-of-fee investment performance.

BvB extend BG by focusing on the empirical implications of fund manager value creation. They argue that, not only is net-of-fee alpha uninformative about fund manager skill, but so is gross-of-fee alpha. Rather, what matters are the economic rents created as measured in dollar terms. To make their point, BvB provide an example (which we modify slightly) of a \$10 billion fund that generates a 1.0% gross-of-fee alpha. Gross economic profits or rents in this case equal \$100 million, which is more socially valuable than \$100 million dollar fund that generates a 10.0% gross-of-fee alpha with gross economic profits or rents of \$10 million. The more socially valuable fund is 100 times larger than the less valuable fund, with the gross of fee alpha declining due to diseconomies of fund scale. The more skilled fund manager further generates ten times the fees relative to the less skilled fund manager, at \$100 million versus \$10 million in management fees. In the end, BvB argue that fund size and management fees are reliable measures of fund manager skill (which is not directly observable on its own).

The primary intuitions from BG and BvB, which are specifically generated in the context of mutual fund investment, have been transported over to PE with little critical examination of the micro-founded differences between the two markets. Most importantly, neither BG nor BvB incorporate search and matching frictions into their analysis (nor should they given how the mutual fund market works), whereas costly search and matching that occurs between LP investors and GP fund managers is of first-order importance in PE.

This is where we come in. To further motivate our search and matching modeling framework, consider the following stylized facts. First, although it is well known and fully accepted that risk-adjusted net-of-fee performance in PE is persistent (e.g., Kaplan and Schoar (2005), along with many others), with positive net-of-fee alpha often observed in the data, there has not yet been a micro-founded explanation for why fund managers fail to fully extract the rents they create in the form of management fees.

Second, according to BvB, there is a monotonically positive relation between fund size as a proxy for fund manager skill and gross-of-fee fund profitability (defined as fund size times gross-of-fee alpha). But BvB predicts no such relation for net-of-fee profitability, since fund managers extract all of the rents they create in management fees. Table 1 shows that, for Buyout and Real Estate funds, not only is there positive net-of-fee fund profitability (defined as fund size times net-of-fee alpha) on average across fund size quintiles, but that net-of-fee fund profitability is increasing in fund manager skill as proxied by fund size. Further note the lack of monotonicity in alpha as a function of fund size, which highlights the fact that profitability, and not alpha, is what matters economically.

Figure 1 Here

Third, in both BG and BvB, capital supplied by investors is infinitely elastic, flowing to fund managers to the point where investors expect to earn returns equal to their outside option of investing in the liquid benchmark. But in search and matching markets like that in PE, not only can market supply and demand conditions vary over time and across fund managers (based, for example, on the measure of fund managers competing within a given sub-market), but investors also need to be compensated for search and matching frictions in PE. This creates a role for "market tightness," which has not been considered as such in the PE literature to date, as an equilibrating mechanism that accounts for capital supply and demand in general equilibrium.

With these facts and relations in mind, we develop a model that extends BG and BvB to account for how investment markets actually work and clear in Private Equity. In doing so we adopt the directed search framework of Guerrieri et al. (2010). That framework incorporates LP-GP matching *rates* on the extensive margin as well as matching *results* on the intensive margin, with equilibrium determined under both complete and asymmetric information regarding fund manager skill.

In the model, as a starting point, the outside investment option for the LP is a liquid security in which expected utility depends on the quantity of capital allocated to the investment, the expected return on investment, and the disutility associated with observing return volatility that occurs between the security purchase and sale dates. This outside option provides a benchmark and represents the opportunity cost of investing in PE relative to a public market equivalent.

In PE, fund managers differ in their skill levels, with skill level groups mapping into sub-markets. Holding fund size constant, skill is defined as fund manager ability to generate pre-fee investment returns in excess of expected returns associated with the liquid benchmark. This excess expected return is also labeled as gross-of-fee alpha. As in BG and BvB, there are diseconomies of scale that erode alpha as fund size increases. For a given fund size, skill level thus hierarchically orders GP sub-market. However, as fund size varies in equilibrium, alphas realized by less skilled fund managers can exceed those realized by more skilled fund managers. In fact, in the model as well as in the data, we see that it is often the case that less-skilled fund managers realize higher alphas than more skilled fund managers. What matters to LP investors, and what is the consequence of fund manager skill, is not alpha *per se*, but how much value is created relative to the outside investment option, as it depends on fund size, alpha as a function of fund size, and GP management fees.

In addition to fund diseconomies of scale and management fees, as well as the potential for asymmetric information regarding fund manager skill, there are two distinct frictions that exist in PE markets that erode LP investor utility relative to the liquid outside option. First, there is an entry fee that LPs must pay to enter the PE market. This can be thought of as the cost of assembling a specialized team of investment and due diligence professionals that go along with investing in private market alternatives. Second, as noted earlier, there is the potential for match failure, the probability of which is determined in equilibrium when failure occurs. It results in the LP investor redirecting its allocated investment amount back into the liquid outside investment option.

Offsetting these costs is one potential benefit to investing in PE relative to the liquid benchmark security, which is a reduction in observed investment return volatility. This reduction is often referred to as NAV smoothing (Brown et al., 2021; Ercan, Kaplan and Strebulaev, 2025). NAV smoothing may generate benefits related to, for example, institutional investor financial reporting or agency/fiduciary responsibilities.

In the full information case, to attract GPs from a particular sub-market, LPs will post a contract specifying the investment quantity, or more simply, the desired fund size. GPs that populate a particular sub-market will then direct their search to a particular LP investor depending on which contract maximizes expected management fees net of fund production costs. Fund production costs increase at an increasing rate as a function of fund size, and are only incurred when a LP-GP match is successful. Otherwise, conditional on match failure, the GP exits the PE market with a zero payoff.

A key measure that characterizes supply and demand conditions by sub-market is "market tightness." As noted, once a decision has been made to enter the PE market, LPs post contracts to screen GPs based on their skill. GPs, aware of their own skill, strategically select which contract to pursue. This results in submarkets with different ratios of LPs posting contracts to GPs seeking funding. This in turn affects the probability of a successful match, which not only varies by sub-market, but also by LP versus GP.

We show that, in equilibrium in which LP capital is supplied elastically until the expected utility of PE investment equals that of investing in the outside option, there is a pecking order of fund size, net-of-fee fund profitability, and market tightness that all increase in fund manager skill. When the benefits of NAV smoothing in PE are sufficiently small, we show that net-of-fee alphas are persistently positive. This is in contrast to BG and BvB, who predict zero net-of-fee alphas on average. The key intuition for understanding the difference largely derives from the "dogs that don't bark" in the PE data. Not only do PE investors need to be compensated for PE market entry costs, but also for the costs of a failed match in which the entry costs are sunk, particularly as failed matches vary across sub-markets. Those failed matches are not seen in the data of PE investment performance – only the successful matches. LP investors realize positive alphas on average in order to offset losses associated with failed matches that are not directly observed in the PE performance data.

There is more investor demand for higher-skilled fund managers, which results in larger fund sizes. As in BvB, the larger fund sizes can erode gross-of-fee alpha to the point where, in equilibrium, it is lower with more skilled fund managers than less skilled fund managers. But, gross-of-fee profitability is higher for more skilled fund managers. In our model, so is net-of-fee fund profitability. This again happens because of the deal dogs that don't bark. Market tightness is higher due to the greater demand for funds produced by more skilled fund managers, which increases the probability of match failure for LP investors. As a result, net-of-fee fund profitability increases to offset the costs associated with a higher rate of match failures.

Thus, under full information, our model shows how differences in GP skill levels naturally segment fund managers into separate market tiers, where higher-skilled GPs command greater management fees due to larger fund sizes, experience greater market tightness, and achieve superior net-of-fee profitability. Our explanation of variation in net-of-fee profits is a foundational contribution of this paper. Unlike Berk and Green (2004) and Berk and Van Binsbergen (2015), who predict uniform net-of-fee returns across fund manager skill types, our model reveals persistent differences in net-of-fee profits across submarkets segmented by GP skill.

We further show that asymmetric information further influences market equilibrium through adverse selection, requiring LPs to design contracts that incentivize GPs to reveal their true skills while deterring imitation by lower-skilled managers. Pecking order relations remain as in the full information case, with higher-skill fund managers having to further increase fund size in order to successfully separate themselves from lower-skill fund managers.

We utilize Preqin data to test comparative static model implications. We provide the first empirical measure of market tightness in PE by using the subscription ratio reported at the fund level. Net-of-fee fund alphas are measured following the methodologies of Gredil et al. (2023) and Korteweg and Nagel (2024), with a new imputation technique introduced for GPs lacking cash flow information (Li and Riddiough, 2024). Empirical results all support comparative static model predictions.

This paper makes several key contributions to the literature. To the best of our knowledge, we are the first to investigate the role of search frictions in LP-GP matching. Building on Guerrieri et al. (2010), this paper extends their theoretical framework to a PE context and empirically tests its implications. The model also expands on Guerrieri et al. (2010) by incorporating a risky outside option (public market equivalents) into the general modeling framework.

Secondly, this study offers a micro-founded contribution to the private equity literature by identifying specific forms of illiquidity premia. The first component is a fixed premium reflecting LPs' search costs, a consistent burden due to the inherent illiquidity and informational constraints of the private equity market (Phalippou and Gottschalg, 2009; Franzoni et al., 2012). The second component stems from sub-market-specific risks of match failures between LPs and GPs, adding an additional layer of illiquidity (Axelson et al., 2010; Korteweg and Sørensen, 2017). By distinguishing these forms, this paper clarifies how illiquidity influences PE market dynamics and provides a nuanced perspective on its underlying sources.

Thirdly, existing studies on private equity have not recognized the implications of failed matches on PE investment performance measurement. Not only can we explain, in a rational framework, persistently positive net-of-fee alphas with the full extraction of expected economic rents by fund managers, but we also generate a pecking order of relations that follow from variation in fund manager skill level combined with a failure to match.

Finally, this paper offers an innovative way of framing the "volatility veil" puzzle observed in institutional investments (Barber and Yasuda, 2017; Andonov et al., 2018; Brown et al., 2019; Offodile II et al., 2021; Riddiough, 2022). By incorporating smoothed returns into the investor's mean-variance utility function, we show how return smoothing can distort market equilibrium, possibly to the point where negative average alphas are obtained when they otherwise would be positive.

The remainder of the paper is structured as follows. Section I presents the directed search model. Section II introduces empirical implications of the theory. Section III shows data and measures of alpha and market tightness. Section IV provides empirical testing of equilibrium results. Section V concludes with a discussion of the findings.

I. Directed Search and Matching Between LPs and GPs

I.A. Model Structure

Following Guerrieri et al. (2010), in our model, the LP investor is the principal and the GP fund manager is the agent. There are two points of time: t=0, which is when investment occurs, and

t=1, which is when the investment is liquidated and payouts occur. In between t=0 and t=1, investment performance can be observed by LP investors, but only imperfectly in the case of PE.

At t=0, LPs are interested in allocating money to invest in PE. For investment to occur, LPs must satisfy a participation constraint that requires expected utility from PE investment, inclusive of all fees and related costs, to at least equal the expected utility from investing in the liquid benchmark security.

The expected utility associated with investing in liquid security is also referred to as the LP's *outside option*, formally expressed as follows:

$$V_B = y\left[\mu_B - \frac{\psi}{2}\sigma_B^2\right] > 0 \tag{1}$$

with the B subscript indicating the liquid benchmark.[2] In addition, y>0 denotes the investment amount, μ_B the expected return from investment, and σ_B^2 the volatility of interim returns. The parameter $\psi \geq 0$ quantifies the disutility associated with observed return volatility that is expected to occur between t=0 and t=1.[3]

Now consider the LP's PE investment option. In this case the LP considers a class of investment opportunities that provide the same risk-adjusted expected return and true underlying variance in returns as the liquid security. But, interim returns are imperfectly observed and generally smoothed.

GP fund managers differ in their skill levels, which is their ability to generate pre-fee returns in excess of risk-adjusted return, μ_B. More specifically, suppose there are N different GP types,

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[2] Note that expected utility depends on incremental investment and not total investment effects.
[3] To simplify and to avoid unnecessary complication, we assume the temporal discount rate from t=0 to t=1 is zero. This allows us to ignore discounted differences between the initial investment y at t=0 and the return of that initial investment that is realized at t=1.

i=1,…,N, with higher skill types corresponding with higher index numbers. Define $\alpha_i(y)$ as the measure of GP skill, $\alpha_i(y) > 0$ and continuous for all $0 < y < \infty$. Alpha is persistent and independent of state contingent outcomes that result from μ_B and σ_B^2. There is a strict monotonicity in GP type, in the sense that $\alpha_j(y) < \alpha_i(y)$ for any given *y* with *j*<*i.* Further, $\alpha_i(y)$ is strictly decreasing and convex in *y* to account for diseconomies of scale in fund size, with $\alpha_i(y)$ approaching zero as fund size becomes arbitrarily large. Finally, we require that $\frac{d\alpha_j(y)}{dy} \leq \frac{d\alpha_i(y)}{dy}$, for any given *j*<*i*. This restricts alpha, as a function of fund size, to decline no faster for higher-skill fund managers than for lower-skill fund managers. We will sometimes refer to the GP type as a PE sub-market.

There are three costs to PE investment that the LP must bear. First, to enter into the PE market the LP must pay a fixed cost of *k.* This cost is independent of the exact size of the (endogenously chosen) investment amount, and can be thought of as the cost of assembling a specialized team of investment and due diligence professionals capable of executing all of the necessary complex tasks that accompany PE investment. This cost is sunk once the LP decides to enter the PE market.

Second, because PE is a search and matching market, once the LP enters the PE market it will post a contract specifying its desired fund size. Based on this posted contract, GP fund managers, who differ in their skill levels, will direct their search to the desired contract and hence LP. In equilibrium, Guerrieri et al. (2010) show the posted contract is unique with respect attracting a particular GP type. Thus, there will be N different contracts posted in equilibrium, creating N different sub-markets that correspond to GP type. Search and matching are bilateral, meaning that LPs endogenously choose to post a particular contract, with a one-to-one match occurring with a particular GP type.

Once the initial match occurs, for idiosyncratic reasons there is a probability, $1 - \eta_i(\theta)$, that the match will fail for the LP. The possibility of match failure is a cost to participating in the PE market. Match failure depends critically on the tightness of the sub-market, θ, as will be defined shortly. If the match fails, the LP redirects the allocated investment amount, y, into its outside option, the liquid security.

Third, GPs charge management fees in proportion to fund size, with the proportional management fee denoted by $\phi, 0 < \phi < 1$. Offsetting these costs is realizing $y_i\alpha(y_i)$, the expected (pre-fee) value added from investing in the PE market relative to the outside option of investing in the liquid security.

An additional potential benefit to investing in PE is that the LP investor only imperfectly observes return volatility that is realized from t=0 to t=1. The effect is related to NAV smoothing that is known to occur in PE, where the volatility of interim reported returns from fund investment is smoothed relative to the true return volatility realized from investment in the benchmark public market equivalent (Brown et al., 2019; Jackson et al., 2023; Ercan, Kaplan and Strebulaev, 2025). This phenomenon has also been referred to in the literature as "volatility laundering" (Asness, 2020). Institutional investors, particularly pension fund investors, seem to value imperfect accounting information that smooths interim returns, possibly for performance reporting or regulatory reasons. In this sense, imperfect return observability potentially generates a non-pecuniary benefit for LP investors. We recognize this effect by introducing the disutility parameter, $\psi', 0 \leq \psi' \leq \psi$, that is specific to PE investment relative to investment in the liquid security.

With these specified costs and benefits, the LP's expected utility to enter the PE market can now be formally expressed, as follows:

$$V = \eta(\theta)y\left[\mu_B + \alpha(y) - \frac{\psi'}{2}\sigma_B^2 - \phi\right] + \big(1-\eta(\theta)\big)y\left[\mu_B - \frac{\psi}{2}\sigma_B^2\right] - k \qquad (2)$$

with the sub-market index, i, suppressed for the time being. Expected utility from PE investment, V, is the expected utility from PE investment conditional on a successful match, plus expected utility from pursuing the outside investment option conditional on an unsuccessful match, less the sunk PE market entry fee. Expected utility associated with a successful match accounts for positive alpha, the disutility of observing (possibly laundered) interim return volatility, and management fees.

Recalling the PE market participation constraint requiring that $V \geq V_B$, with V_B defined in (1), and V defined in (2), after some simplification we are left with the following LP investor PE market participation constraint:

$$\eta(\theta)v(y,\theta) = \eta(\theta)y\left[\alpha(y) + \frac{(\psi-\psi')}{2}\sigma_B^2 - \phi\right] \geq k \qquad (3)$$

Since k and y are positive, for entry by the LP into the PE market to occur, the bracketed term in (3) must be positive at any given y. This eliminates GPs from the market that are unable to deliver sufficiently large alphas to offset associated costs. In this sense, fees and costs to investing in PE act as a screening device with respect to GP entry into the PE market. Further, to ensure utility, and therefore equilibrium, is well-defined throughout, we further require $y\alpha(y)$ to be increasing and concave. This is a weak requirement, consistent with that of Berk and Green (2004) and Berk and Van Binsbergen (2015).

Now consider in more detail the GP side of the PE market. A GP that belongs to sub-market i participates only if it expects to at least break even after earning fees, ϕy, and paying fund production costs, $h(y)$. The production cost function is continuous, increasing and convex in y, with $h(0) = 0$. Higher-skill fund managers also engage in more efficient production, in the sense that $h_j'(y) > h_i'(y), y \geq 0$, for sub-market $j<i$. Lastly, to ensure positive fund sizes in all feasible sub-markets, and therefore the existence of feasible allocations to PE, we require $\frac{d(y\alpha_i(y))}{dy}|_{y=0} > h_i'(0)$ for all i. This again is a weak requirement, and can be thought of as a screening requirement for GPs to enter the PE market.

With this, the GP's expected utility from entering the PE market can be determined, as follows:

$$U(y, \theta) = \mu(\theta)u(y, \theta) = \mu(\theta)[\phi y - h(y)] \tag{4}$$

where, as with LP utility, we suppress i for now. Feasible allocations require the bracketed term in (4) to be positive for some y>0, with $h'(0) < \phi$.[4] Expected GP utility (profit) is fee revenue less fund production cost as they depend on fund size, multiplied by the probability of a successful GP match, $\mu(\theta)$. The probability of a successful match depends on market tightness, θ, that in part depends on the measure of GPs available to participate in a given sub-market. An unsuccessful match leads to the GP exiting the PE market with zero profit.

As noted, there is a fixed measure of GPs that differ by type. The total measure of GPs across all sub-markets is normalized to 1.0, with a known fixed proportion of GPs in each sub-market. The proportion of LPs that enter and post contracts in each sub-market relative to GPs that attempt to match by offering fund investment defines sub-market tightness. Sub-market tightness is, as

[4] Once again, this requirement can be thought of as a GP screening condition, where, collectively, satisfying the three noted requirements are necessary as well as sufficient for the GP to gain initial entry into the PE market.

noted earlier, expressed by θ_i. Recalling that the LPs match probability is $\eta(\theta)$, the GP match probability is defined as:

$$\mu(\theta) = \theta\eta(\theta) \quad (5)$$

LP and GP match probabilities are assumed to be well-defined continuously differentiable functions, with the probability of an LP (GP) match, $\eta(\theta)$ ($\mu(\theta)$), strictly decreasing (increasing) and convex (concave) in market tightness, θ. Market tightness as measured by θ is continuous, ranging from [0,∞]. A common example of well-defined market tightness parameters is $\mu(\theta) = \frac{\theta}{1+\theta}$ and $\eta(\theta) = \frac{1}{1+\theta}$.

While there is a finite, known measure of GPs that exist and that are qualified to enter within each sub-market, investment demand in PE is infinitely elastic with respect to LP participation. In other words, within any given sub-market, LP investors will pay the entry cost, *k,* and continue to enter the market as long as $V > V_B$ (expected utility to PE investment exceeds expected utility to investing in the liquid security). Free entry will thus endogenously increase market tightness, and continue until $V = V_B$.

We now have the model structure necessary to determine general market equilibrium under conditions of full information about GP type. The full information case establishes first-best, efficient allocations. Once equilibrium in this environment is characterized, we augment the model structure by introducing the potential for adverse selection based on asymmetric information regarding GP type.

I.B. Equilibrium Under Full Information Regarding GP Type

With the described model structure, we will now define equilibrium and characterize the resulting outcomes.

First, we note that the three necessary assumptions as stated in Guerrieri et al. (2010, section 2.2) on preferences are satisfied. The Monotonicity assumption is satisfied based on the ordering of $\alpha_i(y)$. The Local Nonsatiation as well as Sorting assumptions are satisfied based on the stated model structure under full information.

Equilibrium is defined as in Guerrieri et al. (2010, section 2.4), with: i) LPs posting a contract specifying fund size under free entry, where, in our application, expected utility is maximized when taking the outside liquid security investment option into account; ii) GPs direct their search based on the posted contract in order to maximize expected profits from participating in the PE market; and iii) Market clearing with active participation by GPs within a given sub-market depending on the exogenous measure of GPs that are qualified to enter the market, as well as the endogenously determined market tightness that, with bilateral trade, results in a certain proportion of GPs exiting the market to earn zero profits. Failed matches will also result in a certain proportion of LPs (that in general differs from the GP proportions) shifting their investment allocation from PE to the liquid outside option.

LPs further recognize that, in equilibrium, within sub-market, GPs optimize production and therefore expected profits based on their endowed skill, knowing that the LP's participation constraint binds with free entry. GPs then direct their search, sorting and matching into the appropriate sub-market. Equilibrium allocations are specifically determined with GP in sub-market i assuming a binding participation constraint in (3), and then replacing $\eta(\theta)$ with $\frac{\mu(\theta)}{\theta}$ (see equation (5)). Total management fees, ϕy, are isolated in (3), with the equivalent terms substituted in to equation (4). The result is expected GP utility (profit), $U(y,\theta)$, equal to:

$$U(y,\theta) = \mu(\theta)\left[y\alpha(y) + \frac{y(\psi-\psi')}{2}\sigma_B^2 - h(y)\right] - k\theta \quad \text{(4a)}$$

In optimizing (4a) with respect to y and θ, observe that the FOC for optimal fund size is independent of market tightness. This separation result greatly simplifies the analysis. Then, given the first-best fund size, market tightness that satisfies the free entry condition is determined.

Theorem 1 characterizes equilibrium outcomes as they vary across sub-markets.

<u>Theorem 1</u>: Equilibrium exists, with optimal y^* and θ^* uniquely determined in each sub-market. Under full information regarding GP type, optimal fund size is increasing in GP skill across sub-markets, as is market tightness.

<u>Proof</u>: See Appendix A

Because of the strict ordering of fund manager skill by type, and given $\frac{d\alpha_j(y)}{dy} \leq \frac{d\alpha_i(y)}{dy}$ for any $j<i$, higher-skilled fund managers can, in equilibrium, produce larger investment funds than less-skilled fund managers. Starting with Berk and Green (2004), a positive causal relation between GP skill level and fund size is well documented in the PE literature. Higher GP skill leading to larger fund size implies additional LP entry, which increases market tightness. Measuring market tightness in the cross-section as we do is new to the PE literature. There is an empirical literature that focuses on time-varying fund inflows, available commitments ("dry powder"), invested capital and investment performance (see, e.g., Chung et al, 2012), but, to our knowledge, there has been no research examining cross-sectional heterogeneity, by sub-market, in market tightness as a function of GP skill level

We now characterize expected net-of-fee profit from PE investment relative to expected profits from investment in the liquid security. In the special case of no disutility reduction due to interim return volatility laundering ($\psi = \psi'$), in equilibrium expected net-of-fee profits from PE investment always exceed those from investing in the liquid security. There is, moreover, a strict ordering of expected profits by PE sub-market, with expected profits increasing in GP skill level. However, when there is disutility reduction due to interim volatility laundering in PE, these relations can break down in equilibrium, with expected profits to PE investment possibly being less than expected profits to investing in the liquid security. Corollary 1 summarizes the results.

Corollary 1: Denote the breakeven fund size in sub-market i as $\hat{y}_i, 0 < \hat{y}_i < \infty$, and define breakeven such that $\alpha_i(\hat{y}_i) - \phi = 0$. $\hat{y}_i$ exists and is unique. Expected net-of-fee profits to PE investment exceed those in the liquid security when $y_i^* < \hat{y}_i$. Otherwise, profits from investing in the liquid security equal or exceed profits expected in PE. When there is no disutility reduction in PE from interim return volatility laundering – i.e., when $\psi = \psi'$ – it is always the case that expected profits realized in PE investment exceed those realized in the liquid security.

Proof: See Appendix B

Putting liquidity differences aside, extant work either predicts no difference on average in net-of-fee expected returns to PE and the benchmark liquid security (Berk and Green, 2004) or no difference in net-of-fee expected profits (Berk and Van Binsbergen, 2015). The explanation for our relations lies in our search and matching framework. Reference to equation (3), together with separation that results in optimal fund size being determined independently of market tightness, makes the critical relations clear. The bracketed term in (3) only depends on the optimal fund size, y_i^*, and is always positive. When $\psi = \psi'$, it is the case that net-of-fee expected returns and profits are always positive, and exceed expected returns and profits to investing in the liquid security. In this case, revealed preference also results in a strict ordering of expected net-of-fee profits (but not necessarily expected net-of-fee returns) with respect to GP skill level.

Importantly, expected LP utility prior to entering into the PE market is distinct from expected utility conditional on entry and experiencing a successful match. Only the latter is seen in the data, while the former is not, providing new insight into PE investment performance measurement. More specifically, equation (3) makes clear that, across all PE sub-markets, and prior to entry, expected utility accounts for PE entry costs as well as for successful versus unsuccessful matches. But empirically, measurable net-of-fee investment performance relative to the liquid benchmark is conditional on a successful match, as summarized by $y_i^*[\alpha_i(y_i^*) - \phi]$.

Note also that the potential for a reduction in the disutility of interim return volatility in PE investment distorts fund size and decreases measured LP investment performance. This happens because utility-based benefits from laundering return volatility increases demand for PE relative to the liquid security option. If the disutility distortion is sufficiently large, with $\alpha_i(y_i^*) - \phi$ becoming negative, the LP investor becomes willing to accept negative risk-adjusted returns in return for observing less volatile interim returns.

Corollary 2 now formalizes comparative static relations.

<u>Corollary 2</u>: *Optimal fund size is increasing in* $\psi - \psi'$ *and* σ_B^2 *when* $\psi - \psi' > 0$*, but is unaffected by k and* ϕ*. Market tightness is similarly increasing in* $\psi - \psi'$ *and* σ_B^2 *when* $\psi - \psi' > 0$*, while decreasing in k and* ϕ*.*

<u>Proof</u>: *See Appendix C*

These results are generally intuitive. The greater the reduction in LP disutility due to the laundering of interim return volatility, the greater the demand for PE investment. This increase in LP demand results in larger fund sizes and increased market tightness, which in turn has the effect of decreasing expected net-of-fee investment performance. Notably, these effects are magnified for funds holding assets with greater actual interim return asset volatility, σ_B^2.

Optimal fund size is unaffected by entry costs and fund manager fees, and follow because entry costs are sunk, while GP fees are homogeneous in fund size. In the case of entry fees, *k*, although there is no direct fund size effect, LP demand is curtailed nonetheless to result in lower market tightness and therefore a higher probability of LP-GP match survival (as seen in equation (3)).

Similarly, although fund size is also independent of the GP fee percentage parameter, higher fund manager fees make the PE investment option less attractive relative to the outside liquid security investment option to reduce LP demand and therefore market tightness.

I.C. Asymmetric Information Regarding GP Type

Now suppose that the GP skill level is unobservable, in the sense that LPs know the proportion of entry-qualified GPs across sub-markets, but do not know to which sub-market a particular GP belongs. The question is whether LPs can design a contract to enable higher-skill GPs to signal their type, at a cost, in order to fully separate from lower-skill GPs. Given the current model structure, the answer is no. What is required in addition is an incentive compatibility condition applied to lower-skill fund managers that deters them from mimicing higher-skill fund managers. This in turn requires higher-skill fund managers to modify fund production, which affects market tightness and measurable PE investment returns.

The constrained optimization problem is formalized as follows. The LP investor's participation constraint expressed in (3) remains as stated. GP utility, $U_i(y,\theta)$, as expressed in (4a), also remains, where we now explicitly index by sub-market to avoid confusion. $U_i(y,\theta)$ is optimized with respect to y and θ, subject to the following incentive compatibility (IC) constraint:

$$\mu(\theta_i)\left[y_i\alpha_i(y_i) + \frac{y_i(\psi-\psi')}{2}\sigma_B^2 - h_j(y_i)\right] - k\theta_i \leq \overline{U}_j \qquad (6)$$

$j<i$, where $\overline{U}_j$ is optimally determined by GPs of type-j choosing the contract posted by LPs that is targeted to sub-market j.

The equation on the LHS of equation (6) is utility to GPs of type j that try to match with the contract that is posted for GPs of higher type-i. This results in type j GP producing a fund of size y_i, which generates fees of ϕy_i. But type-j GPs must bear their own (relatively higher) production costs of $h_j(y_i)$. The RHS of the inequality is the payoff to the type-j GP when it chooses the contract designed for it by LP investors that enter sub-market *j*.

The problem is solved sequentially, starting with GPs of type-1. In this case, subject to satisfying incentive compatibility, the efficient contract determined under full information is posted by LP investors that enter into sub-market 1. This generates utility of $\overline{U}_1$ for type-1 GPs. Now consider the optimal contract posted for type-2 GPs by LP investors that enter sub-market 2. In order to deter type-1 GPs from trying to match with contracts posted by LPs targeting type-2 GPs, fund size and thus market tightness may need to change. Whether or not the contract requires modification depends, in general, on the specific functional forms of α_i and h_i in general. When no contract modification is required, LP investors that target sub-market 2 post the efficient contract. Otherwise the contract is modified to the point in which equation (6) becomes an equality, with indifferent type-1 GPs choosing the contract targeted for sub-market 1. The same process applies to sub-market 3, and so on, until constrained optimal fund size and market tightness are determined in equilibrium across all N sub-markets.

Theorem 2 summarizes how constrained optimal contracting works in the case of asymmetric information relative to the full information case.

Theorem 2: In the case of asymmetric information regarding GP type, a fully separating equilibrium exists in which LP investors targeting sub-market i post contracts that successfully attract only type-i GPs. In equilibrium, fund sizes under asymmetric information equal or exceed fund sizes under full information, with market tightness also equaling or exceeding those under full information. Expected LP investor total profits are higher under asymmetric information, although net-of-fee returns are lower.

Proof: See Appendix D

The intuition for these results is as follows. When incentives exist for lower-skill GPs to defect to mimic higher-skill fund managers, higher-skill fund managers increase fund sizes to the point where lower-skill fund production costs are just high enough to deter defection incentives. Increases in fund size actually increase LP expected profits conditional on a successful match. This happens because, under full information, the first-best fund size is below the point where marginal increases in LP benefits from investing in PE equal marginal increases in fund management fees (see equation (3)). The increase in LP investor demand increases market tightness. Finally, although expected LP profits increase with the larger fund sizes, alpha is lower with the larger fund sizes to decrease returns relative to the full information case.

Interestingly, when sub-markets are sufficiently distinct and heterogeneous, in the sense that there are large differences in GP skill levels or fund production costs across sub-markets, the difference in fund size, market tightness, relative PE profits and returns under full versus asymmetric information are predicted to be small to negligible. However, as sub-markets become less distinct and heterogeneous, it becomes more advantageous for lower-skill fund managers to try to mimic higher-skill fund managers. Relative to the full information case, this case predicts a rightward shift in LP investor demand to increase fund sizes and market tightness, as well as lower returns relative to the liquid security investment option.

II. Testable Empirical Implications

The model generates novel empirical predictions that relate GP skill, fund size, and market tightness. There are also novel predictions that recognize the potential influence of asymmetric information in private equity markets.

II. A. Fund size and Profitability

Similar to predictions following from Berk and Green (2004) and Berk and Van Binsbergen (2015), fund size is predicted to increase with GP skill. The model thus predicts that fund size reveals GP skill, which is obscured in the data by management fees. Furthermore, the model uniquely predicts that incremental net-of-fee profits, as benchmarked to the liquid security, increase with fund size and therefore managerial skill.

II. B. Market tightness increases in fund size

Market tightness, formally defined as the LP-to-GP propensity matching ratio, can be measured based on LP entry into PE markets and by GP fundraising outcomes. Because GP skill and fund size are positively related, with larger fund sizes being associated with greater market tightness, market tightness is predicted to increase with GP skill. Market tightness may be measured empirically by fund subscription rates or time-to-close. The prediction that tighter markets arise in higher-skill segments is novel in the context of PE.

II. C. Asymmetric information leads to even larger fund sizes and tighter markets

When GP skill is unobservable ex ante, LPs need to design a separating contract that induces low-skilled fund managers to self-select into the desired contracts. When sub-markets are insufficiently heterogeneous, in order to deter lower-skilled fund managers from mimicking higher types, these incentive-compatible contracts imply larger fund sizes and greater market tightness relative to the full-information case. Net-of-fee alphas are therefore predicted to decrease, but relative profits are predicted to increase. Empirically, in circumstances where skill is more difficult to identify, such as in markets with first and second fund offerings, we would expect to observe greater heterogeneity in fund sizes and market tightness across sub-markets.

III. Data and Measures of GP Skill and Market Tightness

III.A. Data

This study utilizes Preqin as the primary data source. Preqin offers comprehensive data on private equity performance and other characteristic information, with access enabled through the Freedom of Information Act (FOIA) and its relationships with LPs and GPs.

Among other things, Preqin reports fund absolute performance (net internal rate of return, net IRR) on a quarterly basis. Net IRRs are calculated from fund-level cash flows, reported either by GPs or by LPs. Preqin's performance data aligns with other leading private equity data providers, such as Burgiss and Cambridge Associates (see, e.g., Harris, et al., 2023; Gupta and Van Nieuwerburgh 2021). Given the varying methodologies employed by these providers, it is unlikely that they share identical biases. To ensure the reliability of performance measurement, we restrict the sample to liquidated funds (Chung et al., 2012). Preqin also provides target fund size and realized fund size information, which allows us to generate an empirical measure of market tightness at the fund level.

The final sample covers two types of North American private equity funds with vintage years between 1977 and 2018, including private equity real estate funds and buyout.[5] During this period, private equity real estate funds have the largest number of funds with absolute

[5] Buyouts and private equity real estate share similar fund structure with an average fund life of ten years. We do not include venture capital in this study as it has different structure and high return skewness. We focus on North American funds because they are the largest PE market and share similar regulations.

performance and fund size information (761 funds, about $240.21 billion capital raised), followed by buyout (667 funds, about $517.32 billion capital raised).

III.B. KN-Direct Alpha

The KN-Alpha (2024) (Korteweg and Nagel, 2024) framework provides a refined approach to measuring net-of-fee risk-adjusted performance of private equity funds. Unlike absolute performance metrics such as IRR, the KN-Alpha emphasizes the timing and magnitude of cash flows while aligning them with benchmarks to isolate the fund's abnormal return. The KN-Alpha method is a stochastic discount factor (SDF) asset pricing approach that generalizes the Kaplan-Schoar PME performance measure.

Specifically, the KN-Alpha formulation compares the discounted present value of cash inflows (distributions) to cash outflows (contributions). This relationship is expressed as:

$$\alpha = \sum_{j=1}^{J} \frac{X_j}{R_h^b(j)} - \sum_{k=1}^{K} \frac{C_k}{R_h^b(k)} \tag{1}$$

where X_j represents investor the distributions received at time j, and C_k denotes the investor contributions made at time k. The discount factor, R_h^b, is defined as:

$$R_h^b = \exp\{r_h^f + \beta(r_h^m - r_h^f) - \frac{h}{2}\beta(\beta - 1)\sigma^2\} \tag{2}$$

where r_h^f is the risk-free rate, r_h^m is a vector of factor returns, β is a vector that measures the sensitivity of fund returns to factor returns, and σ^2 is a vector that captures the variance of returns. This formulation ensures that the performance evaluation accounts for broader market conditions and the fund's exposure to systematic risk.

Factor benchmark selection is based on the work of Gupta and Van Nieuwerburgh (2021) and Korteweg and Nagel (2024). We use a one-factor version of the model tailored to capture risks that are specific to the PE asset class For buyout funds, we adopt the large-value index (BH-M) to reflect buyout investments in mature and stable companies. For real estate, we follow the work of Gupta and Van Nieuwerburgh (2021), who show that real estate fund returns primarily load on listed REITs. Accordingly, we use the FTSE NAREIT All Equity REIT Index as the benchmark for real estate funds.

An issue with the KN-Alpha method is that it does not adjust for cash flow duration. This makes it hard to compare benchmarked performance between alternative investments that might have different cash flow timing. Gredil et al. (2023) address this duration issue by asking the following question: What constant number DR (direct alpha), $DR \geq -1.0$, will, as an augment to the benchmark discount rate, generate $\alpha = 0.0$?

$$\sum_{j=1}^{J} X_j * \frac{(1+DR)^{-(h+j)}}{R_h^b(j)} - \sum_{k=1}^{K} C_k * \frac{(1+DR)^{-(h+k)}}{R_h^b(k)} = 0 \tag{3}$$

Equation (3) ensures that the performance is accurately adjusted to reflect both the timing and risk exposure of the fund's cash flows, yielding an annualized abnormal return that is comparable across funds.

To evaluate KN-DR, we require complete fund-level cash flow information, which is highly missing in the Preqin data. To address this issue, we employ the Fund Covariant Regression (FCR)-LASSO imputation method developed by Li and Riddiough (2024). This machine-learning approach estimates direct alphas for funds with missing cash flows using a model trained with funds that have cash flow information.

We follow the same setting in Li and Riddiough (2024), which also uses the Preqin dataset, and which shows that Fund-IRR is the most important factor in estimating alpha. They also show that funds with and without cash flows follow similar distributions in Fund-IRR, resulting in a high estimation precision with little or no bias. The FCR-LASSO model is trained using both liquidated and non-liquidated funds, incorporating 84 covariates to capture various fund characteristics. These covariates include log of Fund-IRR and benchmarked market IRR, the original, quadratics, and cubic terms of fund life, fund sequence number, and fund size, and interaction terms between fund size and year dummies. The model further accounts for potential reporting biases by considering whether the funds are liquidated, whether the GP is publicly listed, whether cash flows are reported for all or some prior funds, and whether the GP follows a single-strategy focus.

Table 1 About Here

Panel A of Table 1 shows the estimated factor loading results across real estate and venture capital. Real estate funds show a moderate beta of 0.747. In the case of buyout funds, the asset-specific beta is 0.461. Panel B of Table 1 shows the performance of the FCR-LASSO model in simulating direct alpha for fund without fund-level cash flow information. The root mean squared error (RMSE) values are 2.809% for real estate and 1.856% for buyouts. These estimates imply that, collectively, the imputed alphas are more than 96% likely to fall within 0.25% of the true values.[6]

Panel C of Table 1 shows final sample sizes for real estate and buyout, with 761 real estate funds and 667 buyout funds included in the analysis. The risk-adjusted KN-DR metric averages 9.602% for buyout funds. In contrast, real estate shows an average of 3.061%. The standard deviation of KN-DR equals 17.91% for buyout and 13.52% for real estate.

III.C. Market Tightness

[6] See Li and Riddiough (2024) for details.

To empirically measure market tightness, we employ the subscription ratio, defined as the realized fund size divided by the target fund size. This ratio is calculated at the fund level. A ratio above one indicates that investor demand exceeds supply, implying tighter market conditions.[7]

Table 2 presents summary statistics for market tightness across different vintage years for private equity real estate and buyout funds. For real estate funds, the mean subscription ratio increased from 1.004 (SD =0.279) prior to 2005 to 1.057 (SD = 0.289) during the 2005–2008 period, before declining to 0.975 (SD = 0.310) for post-2008 vintages. For buyout funds, the average subscription ratio was highest prior to 2005 at 1.070 (SD = 0.331), declining to 0.949 (SD = 0.335) in the 2005-2008 vintages, and then equaling 0.972 (SD = 0.332) for post-2008 vintages. Overall, the results suggest that market tightness is procyclical, with a higher oversubscription rate during pre-GFC vintage years, with reduced market tightness during and after the financial crisis.

Table 2 About Here

IV. Empirical Testing of Search and Matching Model Predictions

In this section, we test the empirical implications outlined in Section II.

IV. A. Relationship between Fund Manager Skill and Fund Size

Our search model framework predicts that net-of-fee profits, defined as Fund Size times net-of-fee Alpha, measure fund manager skill. Tables 3.A through 3.C report relationships among fund

[7] In labor market research, market tightness is commonly measured as the ratio of job vacancies to unemployed workers (Shimer, 2005), where a higher ratio indicates greater demand for labor relative to supply. Our measure serves as an analogous concept in fundraising. Just as a high vacancy-to-unemployment ratio reflects a tight market for employers, a subscription ratio above one indicates excess investor demand relative to fund supply, which implies tighter fundraising conditions.

size, net-of-fee profits, and net-of-fee direct alpha by fund size quintiles and fund sequencing groups within fund managers for the full sample, real estate funds, and buyout funds, respectively. As shown in the full sample, there is a strong positive relationship between fund size and net-of-fee profits. In the full sample (Table 3.A), the average profits increase from $5.41 million in the lowest size quintile (Q1) to $92.18 million in the highest (Q5). This monotonic increase is consistently observed across sequencing groups, which is consistent with the model's prediction that fund size reflects GP skills.

Table 3 About Here

This positive relationship between fund size and profits is also evident within each asset class. For real estate funds (Table 3.B), profits generally increase across size quintiles, though the pattern is less apparent than in the full sample. In most sequences, such as Sequences #3, #4, and #5, there is a clear increase in average profits from Q1 to Q5. For example, in Sequence #4, average profits rise from $2.88 million in Q1 to $87.23 million in Q5. However, in sequence #6, for instance, average profits fall sharply in Q5. Such non-monotonicity is consistent with the findings of Li and Riddiough (2023), who find that after fund sequence #5, real estate returns drop significantly. This may reflect greater heterogeneity in LP preferences and the role of volatility disutility reductions.

It is important to note that, across all asset classes and sequencing groups, direct alpha does not monotonically increase with fund size. Consistent with the arguments of Berk and van Binsbergen (2015), this indicates that KN-DR does not reliably reflect GP skill. For example, in the full sample (Table 3.A), average DR declines from 9.60% in the smallest quintile (Q1) to 4.42% in the largest (Q5), reflecting a decrease in returns to scale. Similar patterns are observed in real estate, where DR often turns negative in higher quintiles despite rising profits. Even in buyouts, where performance sorting is more evident, DR remains flat or peaks in lower quintiles. These findings are consistent with the model's prediction that alpha does not rise with skill, and that fund size and profits, not alpha, are more informative measures of GP ability.

To formally examine the relationship between fund size and net-of-fee profits, Table 4 presents OLS regressions where the dependent variable is fund-level net-of-fee profit. The key explanatory variable is fund size, with controls for lagged direct alpha (KN-DR) of the same fund manager, log fund sequence, and fixed effects. Fund sequence number serves as a proxy for asymmetric information: lower sequence numbers indicate earlier funds in a fund manager's track record and thus greater asymmetric information. Columns (1)-(3) exclude fund sequence fixed effects but include the log of fund sequence number, while Columns (4)-(6) incorporate fund sequence fixed effects but do not include the log of fund sequence number. Vintage fixed effects are included in Columns (3) and (6).

Table 4 About Here

In the full sample (Panel A of Table 4), the coefficient on fund size is positive and highly significant across all specifications, ranging from 0.078 to 0.092. The coefficient on lagged DR is strongly significant, indicating persistence in net-of-fee profits. The addition of vintage and fund sequence fixed effects in Columns (4)-(6) has limited impact on the estimated effect of fund size. In the case of fund sequence, the coefficient is negative but not significant, indicating a limited impact of asymmetric information regarding fund manager skill on net-of-fee profits.

Panel B focuses on real estate funds. The estimated coefficients on fund size remain positive and are statistically significant at the 10% level in most specifications. Notably, the coefficient on the log of fund sequence is negative and statistically significant in Column (1), consistent with the model prediction that earlier sequence funds face greater asymmetric information in fundraising and thus generate higher profit. Panel C shows results for buyout funds. The relationship between fund size and profits is stronger and more significant. The coefficient on fund size remains statistically significant at the 1% level across all columns, with values around 0.08. Lagged DR is also significant. However, the coefficients on fund sequence are not significant, again indicating limited impact of asymmetric information fund manager skill on market outcomes.

IV. B. Relationship between Fund Manager Skill and Market Tightness

Table 5 About Here

Tables 5.A through 5.C present the relationship between fund size and market tightness across quintiles of fund size and sequencing groups, for the full sample (Table 5.A), real estate funds (Table 5.B), and buyout funds (Table 5.C), respectively. In the full sample, tightness increases from 0.84 in Q1 to 1.14 in Q5. This monotonic pattern is consistently observed across almost all sequence groups. For example, in Sequence #2, tightness increases from 0.92 in Q1 to 1.17 in Q5. These results are consistent with the model's prediction that market tightness increases with fund size.

Results are also consistent within each asset class. Real estate funds (Table 5.B) show a general increase in tightness with fund size. For instance, in Sequence #1, tightness increases from 0.81 in Q1 to 1.11 in Q5, and in Sequence #2 from 0.95 to 1.08. On the other hand, buyout funds (Table 5.C) exhibit a particularly strong size-tightness relationship. Overall, tightness rises from 0.80 in Q1 to 1.14 in Q5. Similar trends hold within most sequence groups.

Table 6 About Here

To formally test these relationships, Table 6 presents regressions of tightness on fund size, with the control of lagged direct alpha (KN24-DR), the log of fund sequence, and fixed effects. In the full sample (Panel A), fund size is positively and significantly associated with tightness in all specifications, with coefficients ranging from 0.068 to 0.076. Similar results hold for real estate (Panel B) and buyouts (Panel C). The coefficient on lagged DR is consistently positive and significant, and the log of fund sequence is negative and significant, particularly in buyouts, which supports the prediction that asymmetric information increases market tightness in earlier-stage funds of fund managers.

V. Conclusion

In this paper, we develop a general equilibrium model of search in private equity to examine how institutional investors and fund managers interact within a framework shaped by fund manager skill and search frictions. The model shows that high-skill GPs oversee larger funds and operate in tighter submarkets, achieving higher net-of-fee profits. The presence of adverse selection due to asymmetric information about GP skill alters these outcomes by increasing market tightness and optimal fund size as LP investors modify their strategies to screen out imitators. By accounting for a possible disutility reduction based on the smoothing of reported investment returns, the model further predicts larger fund sizes, higher market tightness, and the potential for negative alphas.

Using novel proxies for GP skill and market tightness, we provide empirical support for the model's core predictions. We find that net-of-fee profits and market tightness increase with GP skill across fund types (as proxied by fund size), where, as predicted, alpha does not reliably measure fund manager skill.

This paper contributes to private equity literature by offering a comprehensive framework that links search frictions, adverse selection, and profits in a single model. It further extends prior work by empirically testing these theoretical constructs in a private equity setting. Our findings provide insights into how private market participants adjust their strategies in response to search frictions and adverse selection.

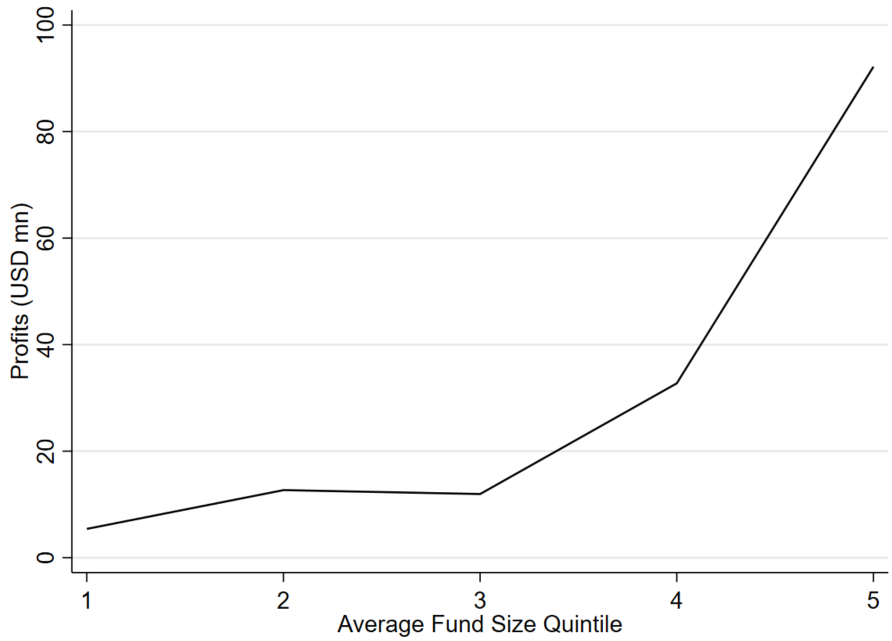

Figure 1: Relationships between fund size quintiles and profits
Note: this figure combines both buyouts and real estate funds.

Table 1: Performance of KN-Alpha Model across Asset Classes

	Real Estate	Buyouts
Panel A: Beta Estimation		
β_{Unique}	0.747	0.461
Panel B: ML-LASSO Performance		
RMSE	2.809	1.856
Confidence level with 0.25% tolerance	96.072%	99.995%
Panel C: KN24-DR Summary		
N	761	667
Average KN24-DR	3.061	9.602
Median KN24-DR	3.577	8.289
SD-KN24DR	13.522	17.911

Note: Panel A presents the estimated CRSP and asset-specific betas across the two asset classes. Panel B reports the RMSE values, along with the confidence levels achieved with a 0.5% tolerance. Panel C summarizes the KN24-DR performance statistics, including the average, median, and standard deviation, as well as the sample sizes for each asset class.

Table 2: Market tightness

Vintage	N	Average	min	median	max	std
Panel A: Real Estate						
<2005	107	1.004	0.240	1.001	1.616	0.279
>=2005 & <=2008	107	1.057	0.251	1.063	2.000	0.289
>2008	32	0.975	0.133	1.016	1.400	0.310
Panel B: Buyouts						
<2005	71	1.070	0.350	1.000	2.500	0.331
>=2005 & <=2008	116	0.949	0.120	1.000	1.827	0.335
>2008	152	0.972	0.225	1.000	2.136	0.332

Note: the market tightness is measured by subscription ratio.

Table 3.A: Full sample profits by sequence and fund size groups

	Overall					Sequence #1				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	274	61.09	5.41	9.60	8.85	107	27.41	4.04	12.26	14.74
Q2	288	169.32	12.20	7.07	7.20	109	76.90	5.54	7.66	7.20
Q3	285	318.73	12.63	4.55	3.96	108	162.27	9.45	5.58	5.82
Q4	288	549.47	32.56	4.95	5.93	109	311.05	19.60	6.41	6.30
Q5 (Highest)	291	1521.94	92.18	4.42	6.06	109	955.09	52.71	6.38	5.52

	Sequence #2					Sequence #3				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	62	50.04	4.37	9.16	8.73	40	81.43	3.71	4.94	4.56
Q2	64	145.79	13.87	9.04	9.51	42	197.33	14.97	6.20	7.59
Q3	63	251.11	16.90	5.58	6.73	42	357.52	22.26	5.63	6.23
Q4	64	448.22	28.28	5.52	6.31	42	622.24	26.20	2.84	4.21
Q5 (Highest)	64	1206.39	33.29	2.02	2.76	42	1632.81	107.80	4.46	6.60

	Sequence #4					Sequence #5				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	21	89.98	6.04	8.48	6.71	12	67.83	9.67	12.97	14.25
Q2	23	202.76	20.44	8.86	10.08	14	274.67	23.94	6.14	8.71
Q3	22	391.31	23.64	2.85	6.04	14	458.08	57.52	8.29	12.56
Q4	23	696.71	55.47	9.30	7.96	14	838.99	66.32	0.86	7.91
Q5 (Highest)	23	1963.12	118.27	5.91	6.02	14	2358.11	248.56	5.65	10.54

	Sequence #6					Sequence >#6				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	9	118.34	6.48	5.26	5.48	23	159.94	14.28	7.52	8.93
Q2	10	265.85	8.86	0.52	3.33	26	446.04	20.56	3.64	4.61
Q3	10	431.98	-4.00	-4.39	-0.93	26	889.83	-29.83	-2.59	-3.35
Q4	10	1049.94	47.07	1.07	4.48	26	1201.97	64.98	1.20	5.41
Q5 (Highest)	11	3811.16	146.02	-2.25	3.83	28	2603.80	236.27	3.00	9.07

Note: Table 3.A reports summary of both real estate and buyouts by fund size quintiles across different sequencing groups. Within each sequence, funds are sorted into quintiles based on size. For each quintile, the table shows the number of funds, average fund size, average profit, the simple average direct alpha (DR), and the fund-size-weighted average DR. Quintile 1 includes the smallest funds, and Quintile 5 the largest. The weighted DR is calculated using fund size as weights.

Table 3.B: Real Estate profits by sequence and fund size groups

	Overall					Sequence #1				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	146	32.11	1.99	6.14	6.21	49	17.06	0.69	4.77	4.07
Q2	154	90.48	3.72	4.84	4.11	50	47.62	2.36	5.48	4.96
Q3	152	179.72	1.10	1.80	0.61	50	116.66	3.85	4.01	3.30
Q4	154	334.24	5.99	2.87	1.79	50	252.56	14.45	5.56	5.72
Q5 (Highest)	155	921.27	3.94	-0.18	0.43	50	549.54	1.76	1.12	0.32

	Sequence #2					Sequence #3				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	32	28.50	2.56	8.90	8.97	22	33.28	2.50	6.32	7.53
Q2	33	75.33	4.38	6.03	5.82	23	85.71	4.59	4.77	5.35
Q3	33	147.13	2.63	2.16	1.79	23	167.98	7.94	4.42	4.72
Q4	33	267.70	11.03	3.92	4.12	23	356.85	0.87	0.39	0.24
Q5 (Highest)	33	714.82	-12.65	-2.29	-1.77	23	1069.54	25.52	1.05	2.39

	Sequence #4					Sequence #5				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	12	34.93	2.88	6.99	8.24	8	55.76	0.98	3.40	1.75
Q2	13	88.51	6.12	6.63	6.91	9	157.97	5.61	3.32	3.55
Q3	12	162.62	-1.24	-0.19	-0.77	9	265.48	5.50	1.94	2.07
Q4	13	306.67	29.69	10.54	9.68	9	432.33	-30.41	-6.25	-7.03
Q5 (Highest)	13	1263.54	87.23	5.61	6.90	9	1110.06	42.91	1.00	3.87

	Sequence #6					Sequence >#6				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	6	51.07	2.42	4.45	4.74	17	60.93	3.72	5.94	6.11
Q2	7	138.83	-1.37	-2.39	-0.99	19	186.95	4.45	3.33	2.38
Q3	7	280.69	-29.20	-9.40	-10.40	18	358.88	-6.92	-2.78	-1.93
Q4	7	557.50	-15.26	-2.57	-2.74	19	527.51	-9.98	-1.96	-1.89
Q5 (Highest)	7	1548.67	-148.30	-7.92	-9.58	20	1493.69	-6.44	-2.92	-0.43

Note: Table 3.B reports summary of real estate by fund size quintiles across different sequencing groups. Within each sequence, funds are sorted into quintiles based on size. For each quintile, the table shows the number of funds, average fund size, average profit, the simple average direct alpha (DR), and the fund-size-weighted average DR. Quintile 1 includes the smallest funds, and Quintile 5 the largest. The weighted DR is calculated using fund size as weights.

Table 3.C: Buyouts profits by sequence and fund size groups

	Overall					Sequence #1				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	128	94.15	9.30	13.55	9.88	58	36.15	6.87	18.58	19.00
Q2	134	259.93	22.60	9.97	8.70	59	101.72	8.23	9.51	8.09
Q3	133	477.60	24.15	7.10	5.06	58	201.59	14.32	6.96	7.10
Q4	134	796.82	64.07	7.62	8.04	59	360.62	23.93	7.12	6.63
Q5 (Highest)	136	2206.54	192.76	9.66	8.74	59	1298.77	95.90	10.84	7.38

	Sequence #2					Sequence #3				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	30	73.01	6.30	9.43	8.64	18	140.27	5.18	3.25	3.69
Q2	31	220.79	25.70	12.83	11.64	19	332.44	27.54	7.94	8.28
Q3	30	365.48	30.80	8.75	8.43	19	586.96	39.60	7.10	6.75
Q4	31	640.40	46.65	7.23	7.29	19	943.51	56.87	5.80	6.03
Q5 (Highest)	31	1729.67	82.20	6.61	4.75	19	2314.66	207.40	8.58	8.96

	Sequence #4					Sequence #5				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	9	163.39	10.25	10.45	6.28	4	91.98	27.05	32.11	29.41
Q2	10	351.29	39.06	11.76	11.12	5	484.74	56.93	11.21	11.74
Q3	10	665.75	53.51	6.49	8.04	5	804.76	151.16	19.72	18.78
Q4	10	1203.77	88.98	7.68	7.39	5	1570.96	240.44	13.66	15.31
Q5 (Highest)	10	2872.57	158.62	6.29	5.52	5	4604.60	618.73	14.02	13.44

	Sequence #6					Sequence >#6				
Group	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	weighted average DR(%)	N	Fund Size ($mn)	Profit ($mn)	Simple average DR(%)	Weighted average DR(%)
Q1 (Lowest)	3	252.90	14.61	6.88	5.78	6	440.47	44.19	12.00	10.03
Q2	3	562.23	32.73	7.31	5.82	7	1149.29	64.28	4.48	5.59
Q3	3	785.00	54.80	7.29	6.98	8	2084.46	-81.39	-2.17	-3.90
Q4	3	2198.97	192.50	9.56	8.75	7	3032.67	268.44	9.79	8.85
Q5 (Highest)	4	7770.53	661.09	7.68	8.51	8	5379.08	843.03	17.80	15.67

Note: Table 3.C reports summary of buyouts by fund size quintiles across different sequencing groups. Within each sequence, funds are sorted into quintiles based on size. For each quintile, the table shows the number of funds, average fund size, average profit, the simple average direct alpha (DR), and the fund-size-weighted average DR. Quintile 1 includes the smallest funds, and Quintile 5 the largest. The weighted DR is calculated using fund size as weights.

Table 4. Relationships between profits and fund size

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: All						
Fund size	0.078***	0.089***	0.091***	0.079***	0.090***	0.092***
	(0.015)	(0.013)	(0.013)	(0.015)	(0.013)	(0.013)
Lag_DR		1.630***	1.539***		1.589***	1.503***
		(0.395)	(0.419)		(0.380)	(0.407)
Ln(fund sequence)	-6.194	-3.825	-6.969			
	(6.178)	(8.055)	(8.946)			
Constant	-5.279	-26.311**	-27.168	-5.872	-33.188***	-31.237*
	(4.978)	(12.611)	(19.519)	(4.455)	(8.172)	(17.755)
Observations	1,428	842	842	1,428	842	842
Adjusted R-squared	0.236	0.340	0.359	0.248	0.354	0.371
Fund sequence number FE	NO	NO	NO	YES	YES	YES
Vintage FE	NO	NO	YES	NO	NO	YES
Panel B: Real Estate						
Fund size	0.045	0.049	0.050*	0.046*	0.050*	0.051*
	(0.029)	(0.030)	(0.029)	(0.028)	(0.029)	(0.027)
Lag_DR		0.676*	0.438		0.610	0.311
		(0.360)	(0.426)		(0.381)	(0.458)
Ln(fund sequence)	-9.631***	-10.127	-8.715			
	(3.694)	(6.504)	(7.627)			
Constant	-2.335	-4.592	-3.663	-4.432	-16.055*	-20.216
	(6.311)	(11.089)	(10.051)	(6.110)	(9.294)	(14.198)
Observations	761	479	479	761	479	479
Adjusted R-squared	0.078	0.103	0.146	0.109	0.128	0.176
Fund sequence number FE	NO	NO	NO	YES	YES	YES
Vintage FE	NO	NO	YES	NO	NO	YES
Panel C: Buyouts						
Fund size	0.076***	0.093***	0.096***	0.078***	0.097***	0.098***
	(0.019)	(0.016)	(0.015)	(0.020)	(0.015)	(0.015)
Lag_DR		2.203***	2.149***		2.002***	2.042***
		(0.722)	(0.820)		(0.603)	(0.731)
Ln(fund sequence)	15.771	19.707	-0.989			
	(18.250)	(26.234)	(26.948)			
Constant	-5.598	-56.636**	-49.096	-1.294	-43.855***	-47.243
	(8.156)	(28.566)	(36.395)	(6.792)	(14.866)	(30.717)
Observations	667	363	363	667	363	363
Adjusted R-squared	0.248	0.360	0.377	0.301	0.430	0.430
Fund sequence number FE	NO	NO	NO	YES	YES	YES
Vintage FE	NO	NO	YES	NO	NO	YES

Note: Table 4 reports OLS regressions of fund profit on fund size, with additional controls for the lagged direct alpha of the same fund manager, fund sequence, and vintage fixed effects. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 5.A: Full sample tightness by sequence and fund size groups

	Overall				Sequence #1			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	107	79.79	0.84	0.83	36	33.23	0.79	0.80
Q2	117	210.69	0.96	0.94	38	102.44	1.00	1.02
Q3	117	362.77	0.98	1.03	37	179.49	0.92	0.95
Q4	117	677.09	1.06	1.07	38	325.77	1.04	1.05
Q5 (Highest)	127	1682.21	1.14	1.19	39	1079.55	1.12	1.24
	Sequence #2				Sequence #3			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	26	52.48	0.92	0.93	17	84.64	0.91	0.87
Q2	27	167.41	1.02	1.01	19	214.72	0.98	1.02
Q3	28	270.92	1.05	1.12	18	388.82	1.03	1.09
Q4	27	507.83	1.13	1.18	19	571.20	1.04	1.04
Q5 (Highest)	29	1288.28	1.17	1.12	20	1700.81	1.14	1.23
	Sequence #4				Sequence #5			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	9	80.94	0.68	0.75	4	183.65	1.13	1.22
Q2	9	223.86	0.89	0.90	5	261.20	0.91	1.05
Q3	9	356.37	0.91	0.93	6	529.47	0.85	0.94
Q4	9	528.30	0.95	1.04	5	749.60	1.06	1.03
Q5 (Highest)	11	1605.20	1.18	1.19	7	1793.86	1.13	1.17
	Sequence #6				Sequence >#6			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	3	140.57	0.58	0.48	12	221.08	0.79	0.75
Q2	4	239.28	0.88	0.86	15	525.36	0.84	0.81
Q3	4	528.75	1.20	1.08	15	847.96	1.00	1.03
Q4	4	2588.60	0.97	1.01	15	1561.23	1.06	1.06
Q5 (Highest)	5	4572.50	1.09	1.10	16	2942.85	1.11	1.22

Table 5.A reports average market tightness of both Real Estate and Buyout across fund size quintiles and fund sequence groups for the combined sample of real estate and buyout funds. Tightness is defined as the ratio of realized fund size to target size, with "Weighted tightness" representing fund size–weighted averages. Within each sequence group, funds are sorted into quintiles based on fund size, from Q1 (smallest) to Q5 (largest).

Table 5.B: Real Estate tightness by sequence and fund size groups

	Overall				Sequence #1			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	63	43.81	0.87	0.85	19	14.03	0.81	0.82
Q2	68	107.39	0.94	0.89	20	39.55	0.97	0.94
Q3	68	208.16	0.93	0.98	19	106.92	0.85	0.84
Q4	68	346.67	1.03	1.04	20	224.94	1.04	1.08
Q5 (Highest)	72	1031.74	1.14	1.17	20	540.36	1.11	1.08
	Sequence #2				Sequence #3			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	11	88.43	0.95	0.95	10	34.14	1.02	0.99
Q2	12	275.50	0.96	0.97	11	80.62	0.92	0.93
Q3	12	425.80	1.18	1.19	10	174.26	0.94	0.95
Q4	12	758.01	1.23	1.24	11	335.55	1.02	1.01
Q5 (Highest)	13	1994.16	1.10	1.08	11	1255.76	1.16	1.27
	Sequence #4				Sequence #5			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	5	30.04	0.73	0.68	3	38.20	1.09	1.12
Q2	5	82.94	0.96	0.97	4	139.00	0.82	0.78
Q3	5	154.06	0.86	0.87	4	230.50	0.78	0.79
Q4	5	193.84	0.85	0.84	4	412.00	1.07	1.07
Q5 (Highest)	6	1646.87	1.11	1.13	5	959.60	1.10	1.15
	Sequence #6				Sequence >#6			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	2	97.20	0.72	0.72	9	144.56	0.83	0.82
Q2	3	214.27	0.94	0.93	10	279.74	0.82	0.79
Q3	3	476.67	1.40	1.32	11	434.76	0.98	0.99
Q4	3	751.47	0.96	0.99	10	589.65	1.06	1.04
Q5 (Highest)	3	1981.20	1.12	1.24	11	1600.51	1.05	1.11

Table 5.B reports average market tightness of Real Estate across fund size quintiles and fund sequence groups for the combined sample of real estate and buyout funds. Tightness is defined as the ratio of realized fund size to target size, with "Weighted tightness" representing fund size–weighted averages. Within each sequence group, funds are sorted into quintiles based on fund size, from Q1 (smallest) to Q5 (largest).

Table 5.C: Buyouts tightness by sequence and fund size groups

	Overall				Sequence #1			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	44	131.31	0.80	0.82	17	54.70	0.78	0.79
Q2	49	354.04	0.98	0.95	18	172.32	1.03	1.04
Q3	49	577.33	1.06	1.06	18	256.08	1.01	1.01
Q4	49	1135.63	1.10	1.08	18	437.80	1.04	1.03
Q5 (Highest)	55	2533.75	1.14	1.21	19	1647.13	1.13	1.30
	Sequence #2				Sequence #3			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	11	88.43	0.95	0.95	7	156.79	0.75	0.83
Q2	12	275.50	0.96	0.97	8	399.10	1.06	1.05
Q3	12	425.80	1.18	1.19	8	657.01	1.14	1.14
Q4	12	758.01	1.23	1.24	8	895.23	1.07	1.06
Q5 (Highest)	13	1994.16	1.10	1.08	9	2244.76	1.12	1.21
	Sequence #4				Sequence #5			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	4	144.58	0.62	0.77	1	620.00	1.24	1.24
Q2	4	400.00	0.80	0.88	1	750.00	1.25	1.25
Q3	4	609.25	0.97	0.95	2	1127.40	1.00	1.00
Q4	4	946.38	1.09	1.09	1	2100.00	1.00	1.00
Q5 (Highest)	5	1555.20	1.25	1.26	2	3879.50	1.21	1.19
	Sequence #6				Sequence >#6			
Group	N	Fund Size ($mn)	Tightness	Weighted tightness	N	Fund Size ($mn)	Tightness	Weighted tightness
Q1 (Lowest)	1	227.30	0.28	0.28	3	450.67	0.68	0.67
Q2	1	314.30	0.70	0.70	5	1016.60	0.88	0.81
Q3	1	685.00	0.57	0.57	4	1984.25	1.04	1.06
Q4	1	8100.00	1.01	1.01	5	3504.40	1.07	1.07
Q5 (Highest)	2	8459.45	1.06	1.06	5	5896.00	1.24	1.29

Table 5.C reports average market tightness of Buyouts across fund size quintiles and fund sequence groups for the combined sample of real estate and buyout funds. Tightness is defined as the ratio of realized fund size to target size, with "Weighted tightness" representing fund size–weighted averages. Within each sequence group, funds are sorted into quintiles based on fund size, from Q1 (smallest) to Q5 (largest).

Table 6. Relationships between fund size and market tightness

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Full sample						
Fund size	0.075***	0.068***	0.069***	0.076***	0.069***	0.069***
	(0.009)	(0.011)	(0.012)	(0.009)	(0.011)	(0.013)
Lag_DR		0.004***	0.003***		0.003***	0.002**
		(0.001)	(0.001)		(0.001)	(0.001)
Ln(fund sequence)	-0.054***	-0.083***	-0.070**			
	(0.018)	(0.030)	(0.031)			
Constant	0.628***	0.701***	0.698***	0.598***	0.660***	0.651***
	(0.052)	(0.068)	(0.061)	(0.054)	(0.069)	(0.066)
Observations	585	374	374	585	374	374
Adjusted R-squared	0.092	0.131	0.138	0.107	0.137	0.147
Fund sequence number FE	NO	NO	NO	YES	YES	YES
Vintage FE	NO	NO	YES	NO	NO	YES
Panel B: Real Estate						
Fund size	0.072***	0.069***	0.076***	0.069***	0.064***	0.068***
	(0.014)	(0.016)	(0.018)	(0.014)	(0.016)	(0.019)
Lag_DR		0.004***	0.004***		0.003***	0.003**
		(0.001)	(0.001)		(0.001)	(0.001)
Ln(fund sequence)	-0.049*	-0.076*	-0.061			
	(0.025)	(0.042)	(0.045)			
Constant	0.662***	0.717***	0.658***	0.645***	0.711***	0.655***
	(0.067)	(0.086)	(0.090)	(0.070)	(0.093)	(0.098)
Observations	339	221	221	339	221	221
Adjusted R-squared	0.061	0.094	0.096	0.081	0.116	0.105
Fund sequence number FE	NO	NO	NO	YES	YES	YES
Vintage FE	NO	NO	YES	NO	NO	YES
Panel C: Buyouts						
Fund size	0.113***	0.125***	0.107***	0.118***	0.128***	0.118***
	(0.018)	(0.022)	(0.024)	(0.018)	(0.022)	(0.024)
Lag_DR		0.004***	0.003**		0.004***	0.003**
		(0.001)	(0.001)		(0.001)	(0.002)
Ln(fund sequence)	-0.102***	-0.179***	-0.166***			
	(0.030)	(0.048)	(0.050)			
Constant	0.401***	0.396***	0.849***	0.344***	0.257*	0.669***
	(0.103)	(0.121)	(0.141)	(0.108)	(0.134)	(0.156)
Observations	246	153	153	246	153	153
Adjusted R-squared	0.174	0.271	0.337	0.203	0.307	0.394
Fund sequence number FE	NO	NO	NO	YES	YES	YES
Vintage FE	NO	NO	YES	NO	NO	YES

Note: Table 6 reports OLS regressions of market tightness on fund size, with controls for the lagged direct alpha of the same fund manager, fund sequence, and vintage fixed effects. Market tightness is measured by the ratio of realized to target fund size. Standard errors are clustered at the fund sequence level. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

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Appendix

Appendix A. Proof of Theorem 1

To prove Theorem 1, we first take first order conditions of equation (4a) with respect to y:

$$\alpha(y) + y\alpha'(y) + \frac{(\psi - \psi')}{2}\sigma_B^2 - h'(y) = 0$$

Because $\frac{d(y\alpha_i(y))}{dy} > \frac{d(y\alpha_j(y))}{dy}$ and $h_j'(y) > h_i'(y)$ for any $i > j$, this implies $y_i^* > y_j^*$. Higher type GPs gain larger fund size.

Set $f_i(y) = y\alpha_i(y) + \frac{y(\psi-\psi')}{2}\sigma_B^2$. Realized preference implies $f_i(y_i^*) - h_i(y_i^*) > f_i(y_j^*) - h_i(y_j^*)$. Because $f_i(y) > f_j(y)$ and $h_i(y) < h_j(y)$ for any y, then $f_i(y_j^*) - h_i(y_j^*) > f_j(y_j^*) - h_j(y_j^*)$. So, $f_i(y_i^*) - h_i(y_i^*) > f_j(y_j^*) - h_j(y_j^*)$.

Now consider the first order condition with respect of θ:

$$\mu'(\theta)\left[y\alpha(y) + \frac{y(\psi - \psi')}{2}\sigma_B^2 - h(y)\right] = k$$

$f_i(y_i^*) - h_i(y_i^*) > f_j(y_j^*) - h_j(y_j^*)$ implies $\mu'(\theta_i^*) < \mu'(\theta_j^*)$. Since $\mu(\theta)$ is concave, $\mu'(\theta)$ is decreasing in θ. This proves that $\theta_i^* > \theta_j^*$ for any $i > j$.

Appendix B. Proof of Corollary 1

Given the free entry condition, i.e. $\eta(\theta)y\left[\mu_B + \alpha(y) - \frac{\psi'}{2}\sigma_B^2 - \phi\right] + (1 - \eta(\theta))y\left[\mu_B - \frac{\psi}{2}\sigma_B^2\right] - k = y\left[\mu_B - \frac{\psi}{2}\sigma_B^2\right]$, we can get $\eta(\theta)y\left[\mu_B + \alpha(y) - \frac{\psi'}{2}\sigma_B^2 - \phi\right] + (-\eta(\theta))y\left[\mu_B - \frac{\psi}{2}\sigma_B^2\right] = k$. Because $\alpha(y)$ is decreasing in y and concave, there exists a unique breakeven point of $0 < \hat{y} < \infty$ such that such that $\alpha(\hat{y}) = \phi$.

If $\psi' = \psi$, $\eta(\theta)y[\alpha(y) - \phi] = k$. Because $\eta(\theta)$, y, and k are positive, $\alpha(y) - \phi$ must be positive, thus expected profits from PE investments must be larger than the one in public liquid markets.

Appendix C. Proof of Corollary 2

Given the free entry condition, i.e. $\eta(\theta)y\left[\alpha(y)+\frac{(\psi-\psi')}{2}\sigma_B^2-\phi\right]=k$, and $\eta(\theta)$ decreases with θ, $\eta(\theta_i^*)<\eta(\theta_j^*)$, which implies $y_i^*\left[\alpha_i(y_i^*)+\frac{(\psi-\psi')}{2}\sigma_B^2-\phi\right]>y_j^*\left[\alpha_j(y_j^*)+\frac{(\psi-\psi')}{2}\sigma_B^2-\phi\right]$.

The first order conditions of equation (4a) with respect to y implies that y^* is independent of k, ϕ, and θ. From the free entry condition, we can prove that $\eta(\theta)$ increases with k and ϕ but decreases with $\alpha(y)$. Furthermore, if $\psi-\psi'>0$, $\eta(\theta)$ decreases with $\psi-\psi'$ and σ_B^2. Because $\eta(\theta)$ decreases with θ, these imply that market tightness θ decreases with k and ϕ, but increases with excess return $\alpha(y)$, $\psi-\psi'$, and σ_B^2 if $\psi-\psi'>0$.

Appendix D. Proof of Theorem 2

Although the incentive compatibility constraint for a type i GP ($i>1$) may become binding, there are cases where it remains non-binding, and the equilibrium coincides with that under full information. This occurs when managing a larger fund size, y_i, is prohibitively costly for a lower-type GP. Specifically, when

$$\mu(\theta_i^*)\left[y_i^*\alpha_j(y_i^*)+\frac{y_i^*(\psi-\psi')}{2}\sigma_B^2-h_j(y_i^*)\right]-k\theta_i^*\leq\bar{U}_j$$

for all $j\leq i$, the incentive constraint is not binding. The subsequent equilibrium analysis focuses on cases where this condition is violated, providing lower-type GPs with an incentive to deviate when contracts from the full information equilibrium are offered.

To solve the equilibrium under adverse selections, we first start with the GP type $i=1$. For $i=1$, there is no lower type to exclude, thus $y_1=y_1^*$ and $\theta_1=\theta_1^*$. Then, we move forward with the GP type $i=2$. In addition to the maximation condition, we include the incentive capability constraint to disincentivize type 1 fund manager to apply for type 2 fund manager contracts. The maximization problem is expressed as

$$U_2=max_{\theta_2,y_2}\,\mu(\theta_2)\left[y_2\alpha_2(y_2)+\frac{y_2(\psi-\psi')}{2}\sigma_B^2-h_2(y_2)\right]-k\theta_2$$

$$\text{s.t. } \mu(\theta_2)\left[y_2\alpha_1(y_2) + \tfrac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_1(y_2)\right] - k\theta_2 \le U_1^*$$

Type 2 GP problem can be expressed as

$$(\theta_2,\ y_2) = max_{\theta_2, y_2}\, \mu(\theta_2)\left[y_2\alpha_2(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_2(y_2)\right] - k\theta_2$$

$$- \vartheta\left\{\mu(\theta_2)\left[y_2\alpha_1(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_1(y_2)\right] - k\theta_2 - U_1^*\right\}$$

Take the first order condition of y_2, we can get

$$\mu(\theta_2)\left[\alpha_2(y_2) + y_2\alpha_2'(y_2) + \frac{(\psi-\psi')}{2}\sigma_B^2 - h_2'(y_2)\right]$$

$$- \vartheta\left\{\mu(\theta_2)\left[\alpha_1(y_2) + y_2\alpha_1'(y_2) + \frac{(\psi-\psi')}{2}\sigma_B^2 - h_1'(y_2)\right]\right\} = 0$$

$$\vartheta = \frac{\left[\alpha_2(y_2) + y_2\alpha_2'(y_2) + \frac{(\psi-\psi')}{2}\sigma_B^2 - h_2'(y_2)\right]}{\left[\alpha_1(y_2) + y_2\alpha_1'(y_2) + \frac{(\psi-\psi')}{2}\sigma_B^2 - h_1'(y_2)\right]}$$

Represent the contracts in the full information case as (y_i^*, θ_i^*). Then, at y_2^*, the denominator of ϑ is negative, so for ϑ to be positive, the numerator has to be negative too, which implies $y_2 > y_2^*$. Also, because the denominator is more negative, this implies $\vartheta < 1$.

Now, turn to the first-order condition of y_2, we get the following equation:

$$\mu'(\theta_2)\left[y_2\alpha_2(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_2(y_2)\right] - k$$

$$- \vartheta\left\{\mu'(\theta_2)\left[y_2\alpha_1(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_1(y_2)\right] - k\right\} = 0$$

Which implies

$$\vartheta = \frac{\mu'(\theta_2)\left[y_2\alpha_2(y_2) + \frac{y_2(\psi - \psi')}{2}\sigma_B^2 - h_2(y_2)\right] - k}{\mu'(\theta_2)\left[y_2\alpha_1(y_2) + \frac{y_2(\psi - \psi')}{2}\sigma_B^2 - h_1(y_2)\right] - k}$$

Given that $\mu'(\theta_2) > 0$, $y_2\alpha_2(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_2(y_2) > y_2\alpha_1(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_1(y_2)$, $y_2\alpha_1(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_1(y_2) > 0$, $y_2\alpha_2(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_2(y_2) > 0$, and $\vartheta < 0$, $\mu'(\theta_2)\left[y_2\alpha_2(y_2) + \frac{y_2(\psi-\psi')}{2}\sigma_B^2 - h_2(y_2)\right] - k$ must be negative, which implies that $\mu'(\theta_2) < \mu'(\theta_2^*)$. Note that $\mu'(\theta_2^*)\left[y_2^*\alpha_2(y_2^*) + \frac{y_2^*(\psi-\psi')}{2}\sigma_B^2 - h_2(y_2^*)\right] = k$, which implies that $\mu'(\theta_2) < \mu'(\theta_2^*)$. So, $\theta_2 > \theta_2^*$.