I. Introduction

Interest rates, financial leverage, and asset values are among the most important variables in the economy. Many economists, policy makers, and investors believe that these variables may *affect* each other; therefore, manipulation of some variables may cause desired changes in the others. While interest rates are traditionally the variable that the Federal Reserve monitors and tries to influence, research on the recent financial crisis highlights the importance of financial leverage in the economy (see, e.g. Geanakoplos (2009), Acharya and Viswanathan (2011), among others). Recognizing this, Federal Reserve researchers are investigating whether changing requirements for mortgages' loan-to-value ratios based on the economic environment could improve financial stability.[1]

The effectiveness of such policies would depend on how the loan to value ratios and property values exactly interact with each other, as well as whether and how they are endogenously determined. For example, if the two variables are only endogenously correlated and do not affect each other, policies that tries to manipulate the loan to value ratios to affect property values would have little effect. A natural starting point to understand the interactions between the loan to value ratios and property values is to understand their long-run equilibrium relationship. However, the existing literature is virtually silent on this very important issue.

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[1] See the speech by Ben S. Bernanke at the Annual Meeting of the American Economic Association at Philadelphia: http://www.federalreserve.gov/newsevents/speech/bernanke20140103a.htm.

This paper develops a very simple theoretical model in which commercial real estate mortgage interest rates, leverage, and property values are jointly determined. I model commercial real estate for two main reasons. First, commercial real estate constitutes a large portion of the total wealth in the economy. The value of commercial real estate mortgages alone, for example, was about $2.3 trillion at the end of 2013.[2] Second, my model assumes that property values are present values of future cash flows, which is more appropriate for commercial than for residual real estate.

The essence of my model is that mortgage lending is trading: property owners, which are borrowers, trade future cash flows (the debt service) for a lump sum (the loan); and lenders trade the lump sum for the future cash flows. For such a trade to take place, borrowers should value the lump sum more than the future cash flows, and lenders should value the future cash flows more than the lump sum. This suggests that borrowers and lenders have different discount rates for the same cash flow of debt service.

Motivated by this notion, I make three simple and plausible assumptions regarding mortgage borrowers and lenders' discount rates for future cash flows generated by the underlying properties. Ranking each dollar of the expected future cash flow generated by the underlying properties from the safest first dollar to the riskiest last dollar, I first assume that both borrowers and lenders' discount rates for the first n dollars are monotonic increasing functions of n. This is plausible, as discount rates should increase with the risk. Second, I assume that lenders have lower discount rates for the safest/first

[2] Data source is the Financial Accounts of the United States, which is published by the Federal Reserve. See http://www.federalreserve.gov/RELEASES/z1/DEFAULT.HTM

dollar than borrowers, due to lenders' comparative advantages in loan underwriting and debt management. This assumption is reasonable and seems necessary for a loan to be originated – having lower discount rates than borrowers, lenders would value the future cash flow more than borrowers, and thus lenders and borrowers would both gain from trading a lump sum, the loan, for the future cash flow, the debt service, with each other. Third, I assume that property owners have lower discount rates for the total cash flow than mortgage lenders, because owners have comparative advantages in managing and operating properties and thus require lower returns. This assumption indicates that owners value properties more than lenders, which explains why they choose to own.

The three simple assumptions suggest that both owners and lenders' discount rates increase with the amount of cash flow, and lenders' discount rate would start with being lower than but intersect with and finally be higher than owners' discount rate. The intersection point corresponds to market equilibrium. For any amount of cash flow lower than that of the intersection, the present value of the cash flow is higher for lenders than for borrowers, so both borrowers and lenders would gain from a loan that is higher than borrowers' present value and lower than lenders' present value. For any amount of cash flow higher than that of the intersection, the present value is lower for lenders than for borrowers, so no loan will be originated. Assuming perfect competition among borrowers and lenders so both gain zero net present value from the loan, the intersection point uniquely defines the equilibrium mortgage interest rate, which is the equal discount rate for borrowers and lenders, and the equilibrium debt service, which is the amount of future cash flow at the intersection point. The equilibrium loan amount equals the present value of the equilibrium debt service calculated using the equilibrium mortgage interest rate as the discount rate. The equilibrium property value is the present value of the total cash flow generated by the underlying properties calculated using property owners' discount rate. Note that lenders' discount rates play no role in determining property values. The equilibrium leverage, measured with either the loan to value ratio or the debt coverage ratio, is certainly uniquely determined by the equilibrium loan amount, debt service, and property value.

My model has important distinctions from the extensive research on the capital structure of firms, which typically treat firms as price takers, treats the cost of borrowing as exogenous, and focuses on the determinants of leverage. My model also differs from recent theories on the joint determinants of loan interest rates and leverage (see, e.g. Geanakoplos (2009)), which often rely on modeling collateral values and the probability of default. My model seems much simpler and only requires disagreements in property owners and mortgage lenders' discount rates to generate market equilibrium – there is no need to assume asymmetric information, monitoring cost, agency cost, or default and sales of collaterals. Further, my model provides original and testable predictions that haven't been suggested by existing theories.

A comparative statics analysis of my model provides a few predictions regarding how exogenous shocks to the discount rate functions of the lenders and borrowers, which can be naturally interpreted as the supply and demand curves of mortgage loans, would affect the mortgage interest rate, leverage, and property values. First, my model suggests that the equilibrium cap rate, while being endogenous in nature, perfectly correlates with borrowers' discount rate function, which is exogenous in my model. Second, my model predicts that increasing demand for mortgage loans, which corresponds to an upward shift of borrowers' discount rate function and can be captured with an increase in the cap rates, would increase the mortgage interest rate and the loan to value ratio, and decrease the debt coverage ratio. This suggests that leverage is counter-cyclical. Third, decreasing supply of mortgage loans, which corresponds to an upward shift of lenders' discount rate function, would decrease the mortgage interest rate and the loan to value ratio, and increase the debt coverage ratio.

I test the above predictions using quarterly time series of mortgage interest rates, loan to value ratios, debt coverage ratios, and cap rates of apartment, office, industrial, and retail properties from the American Council of Life Insurers (ACLI) database from 1990:Q2 to 2011:Q2, as well as data from the Federal Senior Loan Officer Opinion Survey. I use cap rates to measure the demand for mortgages and use the "net percentage of domestic respondents tightening standards for commercial real estate loans" reported in the Federal Senior Loan Officer Opinion Survey, which I call "Tightening Standard", to measure mortgage supply. My theoretical predictions suggest long-run equilibrium relationships, or co-integration in other words, between three endogenous variables, which are mortgage interest rates, loan to value ratios, and debt coverage ratios, and two exogenous variables, which are cap rates and Tightening Standard. I conduct the co-integration tests using the Engle–Granger two-step method.

The results are generally, but not always, consistent with my theoretical predictions. First, I find that mortgage interest rates are positively correlated with Tightening Standard and cap rates, and the three variables are co-integrated. The positive relationship between interest rates and cap rates is consistent with my predictions but the positive relationship with Tightening Standard is not. As mortgage interest rates may be mechanically correlated with cap rates due to their common component of the risk free interest rate. I redo the analysis using the risk premium of mortgage interest rates and cap rates and obtain robust results. Second, I find that the loan to value ratios are negatively correlated with Tightening Standard and positively correlated with cap rates, which is consistent with my predictions, and the three variables are co-integrated. Further, the relationship between the loan to value ratios and cap rates is very strong: a increase of 100 basis points in the cap rate corresponds to about a increase of about 200 to 300 basis points in the loan to value ratios. This empirical finding sharply contradicts the conventional wisdom that financial leverage is pro-cyclical – at least for commercial real estate mortgages, equilibrium financial leverage seems counter-cyclical. Third, I find that debt coverage ratios are weakly or not correlated with Tightening Standard, which is not consistent with my predictions, and negatively correlated with cap rates, which supports my model. Debt coverage ratios are in fact co-integrated with cap rates.

The main contribution of the paper is the simple model that shows disagreements in property owners' and mortgage lenders' discount rates can jointly determine mortgage interest rates, financial leverage, and property values. The model provides novel and testable predictions on long-run equilibrium relationships among these important variables, and the predictions are generally supported by data. A particularly strong and surprising relationship is the positive co-integration between loan to value ratios and cap rates of commercial real estate, which has not been predicted nor identified, to my best knowledge, in the existing literature.

The rest of this paper is organized as follows. Section II develops the theoretical model and derives testable predictions. Section III summarizes the data and presents empirical results. Our conclusions are presented in the last section.

II. A Theoretical Model of Financial Leverage

To simplify the modeling, I consider a large number of identical properties that generate cash flows that are risky constant perpetuities. I assume that loans are interest-only and have infinite maturity, so debt services are also constant perpetuities. I assume that there are two types of investors in the market – property owners, which are mortgage borrowers, and mortgage lenders, both of which are risk averse. Therefore, ranking each dollar of the expected future cash flow generated by the underlying properties from the safest first dollar to the riskiest last dollar, both borrowers and lenders' discount rates for the first n dollars are monotonic increasing functions of n. I further assume that owners and lenders have different expertise. Property owners (borrowers) have comparative advantages in managing properties: perhaps they can generate higher margins or achieve lower risk in the total cash flows than mortgage lenders. Therefore, they require lower returns (having lower discount rates) for the total cash flow than lenders. Mortgage lenders have comparative advantages in loan underwriting and debt management.

Consequently, they require lower returns and thus have lower discount rates for low-risk portion of the cash flows than property owners.

The essence of the model is that lending is essentially trading – the property owner (borrower) trades a stream of future cash flow, which is the debt service, for a lump sum, which is the loan, with the lender. The net present value of the trading for the owner equals the loan amount minus *the owner's* present value of future debt service. The net present value for the lender is the *lender's* present value of future debt service minus the loan amount. For such a trade to take place, the lender's present value of future debt service needs to be higher for that of the property owner. Any number that is higher than owner's present value and lower than the lender's present value would be an accept loan amount as it give both sides positive net present values.

The property owners' discount rate function can be interpreted as the demand curve for mortgage loans. If the market mortgage interest rate is lower than owners' discount rate, the loan amount, which is the present value of the debt service calculated using the mortgage interest rate as the discount rate, is higher than the present value of the debt service for property owners calculated using their discount rate. Therefore, owners would trade the future debt service for the loan. At the same time, the lenders' discount rate function can be interpreted as the supply curve for mortgage loans. If the market mortgage interest rate is higher than lenders' discount rate, the loan amount would be lower than the present value of future debt service for lenders, which is calculated using lenders' discount rate. As a result, lenders would like to trade the loan amount for future debt service.

Now consider a simple linear specification of the model, in which the discount rates for both owners and lenders are linear increasing functions of income, the total of which is normalized to be $1. Since owners have a higher discount rate for the first penny and a lower discount rate for the whole $1 than lenders, owners and lenders' discount rate functions intersect each other, as Figure 1 illustrates. It is apparent that, for any cash flow level lower than that of the intersection point, owners' discount rate is higher than that of lenders. Consequently, the present value of the perpetual cash flow is higher for the lenders than for the owners. For instance, for the cash flow level corresponding to the vertical line in Figure 2, property owners' discount rate is at point A and lenders' discount rate is at point B. It is clear that this particular level of risky cash flow is worth more for lenders than for owners. Therefore, a loan with the debt service equal this cash flow level would generate positive NPV for both owners and lenders if the mortgage interest rate is between A and B.

For a cash flow level higher than that of the intersection point, as illustrated in Figure 3, owners have a lower discount rate than lenders. For example, in Figure 3, owners' discount rate is at point B and lenders' discount rate is at point A. Since owners' discount rate is lower than that of lenders, the present value of this cash flow is more valuable for owners than for lenders. In this case, no loan would be originated.

Assuming perfect market competition so that the NPV of loans is 0 for both owners and lenders, the interaction point between owners and lenders' discount rate functions would determine market equilibrium. To obtain numeric results, I assume that the owners' discount rate, r_o, is the following linear function of the cash flow (debt service) level x,

$$r_o = a_0 + b_o x \, , \tag{1}$$

where a_o is the intercept term and b_o is the slope. The lenders' discount rate, r_L, is a similar linear function,

$$r_L = a_L + b_L x \; . \tag{2}$$

My assumption about the different comparative advantages of owners and lenders requires that

$$a_O > a_L > 0 \tag{3}$$

and

$$b_L > b_O > 0 \, . \tag{4}$$

The intersection point between the two discount rate functions corresponds to the market equilibrium. I call property owners' discount rate function the *demand curve* of loans and that of lenders the *supply curve*. Note that when the demand curve shifts upwards, the equilibrium debt service and the loan interest rate increase, which is consistent with the impact of increasing demand. Further, when the supply curve shifts upwards, lenders require higher returns for the same amount of cash flow, which corresponds to a decrease in mortgage supply.

This simple model contains seven related endogenous variables – the loan interest rate, the debt service level, the loan value, the property value, the cap rate, the loan to value ratio, and the debt service ratio – all of which are jointly determined by four parameters that characterize owners and lenders' discount rate functions, a_O, b_O, a_L, and b_L. The equilibrium mortgage interest rate, r^*, is as follows,

$$r^* = \frac{a_O b_L - a_L b_O}{b_L - b_O}, \tag{5}$$

which is apparently positive because of (3) and (4). The equilibrium debt service level, x^*, is

$$x^* = \frac{a_O - a_L}{b_L - b_O}, \tag{6}$$

which is also positive. Given the equilibrium interest rate and debt service level, the market value of the loan, L^*, is the present value of the debt service with the equilibrium interest rate as the discount rate,

$$L^* = \frac{a_O - a_L}{a_O b_L - a_L b_O}, \tag{7}$$

which is positive. The capitalization rate of the property, $caprate^*$, is simply owners' discount rate of the whole $1.

$$caprate^* = a_O + b_O \tag{8}$$

The property value, V^*, is the present value of the $1 perpetual income.

$$V^* = \frac{1}{a_O + b_O} \tag{9}$$

The loan to value ratio, LTV^*, can be easily calculated using the loan and property values.

$$LTV^* = \frac{a_O^2 + a_O b_O - a_L a_O - a_L b_O}{a_O b_L - a_L b_O} \tag{10}$$

The debt coverage ratio, DCR^*, is the total cash flow \$1 divided by the equilibrium debt service.

$$DCR^* = \frac{b_L - b_O}{a_O - a_L} \tag{11}$$

Now I analyze the comparative statics of the model, focusing on how shifting of owners and lenders' discount rate functions (changes in their intercept terms) affect the endogenous variables. As both discount rate functions are linear, the partial derivatives of the endogenous variables with respect to the intercept terms are easy to calculate. I present the partial derivatives of four variable observed in my data – the loan interest rate, the loan to value ratio, the debt coverage ratio, and the cap rate – in Appendix A and summarize their signs in Table 1.

All partial derivatives have unambiguous signs, which indicate how the four endogenous variables would react to exogenous shifts in owners and lenders' discount rates. Specifically, when owners' discount rate curve shifts up (a_O increases), the equilibrium loan interest rate increases; the loan to value ratio increases; the debt coverage ratio decreases; and the cap rate increases. It is interesting to note that the partial derivate of the cap rate with respect to a_O is 1, which seems to suggest that the cap rate is a perfect measure of the position of owners' discount rate curve. When lenders' discount rate curve shifts up (a_L increases), the equilibrium loan interest rate decreases; the loan to value ratio decreases; the debt coverage ratio increases; and the cap rate is not affected. These comparative statics are hypotheses I will test in next section.

III. Data and Empirical Analysis

Data used in this paper are from two sources: the American Council of Life Insurers (ACLI) and Federal Senior Loan Officer Opinion Survey on Bank Lending Practices. From the ACLI database, I obtain quarterly time series of mortgage interest rates, loan to value ratios, debt coverage ratios, and cap rates, all of which are averages across mortgages newly originated by life insurers, for each of four property types – apartment, office, retail, and industrial. The ACLI database contains a few other property types, such as hotel/motel; however, the number of loans originated for them tends to be small and there are missing observations so I do not include them in my analysis. Figures 4 to 7 plots the time series of the four variables from 1990:Q2, which is the first quarter for which the Federal Senior Loan Officer Survey data are available, to 2011:Q2.

I obtain two quarterly measures of supply and demand for commercial real estate loans from the Loan Officer Opinion Survey: Net Percentage of Domestic Respondents Tightening Standards for Commercial Real Estate Loans, which I call "Tightening Standard", and Net Percentage of Domestic Respondents Reporting Stronger Demand for Commercial Real Estate Loans, which I call "Stronger Demand". Figure 8 plots the quarterly series of these two time series. Two things are worth noting. First, "Stronger Demand" has a much shorter time period than Tightening Standard and all the ACLI time series, which is undesirable. Second, there seems to be an almost perfect negative correlation, which is -0.84, between Tightening Standard and Stronger Demand. When interpreting Tightening Standard as decreasing supply of mortgages, it seems unlikely that demand would decrease at the exact time and with exactly the same magnitude with the supply. Without knowing how loan officers interpret "demand", I suspect that Stronger Demand does not perfectly measure demand changes; instead, loan officers might be reporting a mixture of demand changes and the *impact* of their tightening underwriting standards, such as fewer loan originations. The correlation between the Tightening Standard and Stronger Demand also creates econometric problems. It becomes difficult to disentangle the impact of supply and demand if the two measures are almost perfectly correlated.

My theoretical model suggests that the cap rate is a perfect measure of the mortgage demand. Note that the partial derivation of the cap rate with respect to the intercept of the discount rate function of property owners is 1. Therefore, changes in the cap rate theoretically perfectly measure shifts in the demand curve: a higher cap rate indicates higher demand for mortgages. Further, the cap rates of the four property types from the ACLI database are all positively correlated with Stronger Demand from the loan officer survey. The correlations range from 0.20 to 0.33. As a result, I use the cap rate instead of Stronger Demand to measure the demand for mortgages. I use Tightening Standard to measure mortgage supply: higher net percentage of loan officers reporting tightening standards indicates upward shifting of the supply curve in my model. Essentially, Tightening Standard is a proxy for a_L in the model.

Table 2 reports the mean, standard deviation, minimum, median, and maximum of the quarterly mortgage interest rate, loan to value ratio, debt coverage ratio, and cap rate of apartment, office, retail, and industrial properties respectively. All summary statistics are very similar across property types. Table 3 reports the correlations between pairs of the mortgage variables. It is clear that the correlations are similar across property types. Further, when considering the cap rate as a measure of the demand curve and the Tightening Standard as a measure of the supply curve, the correlations are usually consistent with the comparative statistics presented in Table 1. For example, the mortgage interest rate is positive correlated with the cap rate; the loan to value ratio is positively correlated with the cap rate and negatively correlated with Tightening Standard; and the debt coverage ratio is negatively correlated with the cap rate. However, a few correlations are inconsistent with my theoretical predictions. For example, the correlation between the interest rate and the Tightening Standard is positive, while the theory suggests a negative partial derivative.

To provide more conclusive results on how three observed endogenous variables – the mortgage interest rates, the loan to value ratio, and the debt coverage ratio – are determined by the demand and supply curves, I conduct a co-integration analysis using the Engle–Granger two-step method on the long-term relationships between the above mortgage variables and Tightening Standard, which I use to measure mortgage supply, and the cap rate, which I use to measure mortgage demand. In the first step, I run a time series regression of each of the three endogenous variables against Tightening Standard and the cap rate respectively and then against both of them. I compare the estimated coefficients with my theoretical predictions in Table 1. In the second step, I use the Augmented Dickey-Fuller Test unit root test to test for unit roots in the residuals of the first step regression. Rejecting the existence of the unit root is evidence for long-run equilibrium relationships.

Table 4 reports results of the time series regressions and the residual-based unit root tests for mortgage interest rates. The focus should be the regressions that include both Tightening Standard and the cap rate as explanatory variables, as the equilibrium mortgage interest rate is affected by both demand and supply of mortgages. These regressions show that the interest rate is positively correlated with both Tightening Standard and the cap rate. The positive relationship between the interest rate and Tightening Standard is inconsistent with my theoretical prediction. The positive relationship between the interest rate and the cap rate, on the other hand, supports my theory. While both relationships are statistically significant, note that the coefficient of Tightening Standard is about 0.01 and that of the cap rate is about 1. Therefore, it seems fair to say that the magnitude of the coefficients seems to favor my theory. Unit root tests on residuals from the time series regressions, which are plotted in Figure 9, significantly reject the existence of unit roots; therefore, the mortgage interest rate is co-integrated with Tightening Standard and the cap rate. When the time series regressions include either Tightening Standard or the cap rate but not both, the coefficient remains statistically significant for the cap rate, but not Tightening Standard.

Note that both the mortgage interest rate and the cap rate contain the "risk-free" interest rate as a common component. Could the positive relationship between the two rates are simply driven by this common component? To investigate this possibility, I redo the regressions and unit root tests in Table 4 using risk premium of the mortgage interest rate and that of the cap rate by subtracting the 10-year Treasury yield from both rates, and report the results in Table 5. It is clear that, while magnitude of the coefficients differs, the signs and the significance levels of the coefficients remain the same. Further, the existence of unit roots is rejected. Therefore, the long-run equilibrium relationships between the mortgage interest rate and the cap rate does not seem to be driven by the risk-free interest rate as their common component.

Table 6 reports results for mortgage loan to value ratios. First, the loan to value ratio is negatively related to Tightening Standard except for apartment, which is consistent with my theoretical prediction. Second, the loan to value ratio is positively related to the cap rate, which also supports my theory. It is interesting to note that this relationship is very strong economically – a 100-basis point increase in the cap rate corresponds to a 186 basis point increase in the loan to value ratio for apartment, a 334 basis point increase for office, a 189 basis point increase for industrial, and a 254 basis point increase for retail properties. Third, the Augmented Dickey-Fuller tests on regression residuals, which are plotted in Figure 10, strongly reject the existence of unit roots and provide evidence for co-integration between the loan to value ratio, Tightening Standard, and the cap rate.

Table 7 report results for debt coverage ratios. There is some but weak evidence that the debt coverage ratio is negatively related to Tightening Standard, which is inconsistent with my theory. The evidence is very strong for a negative relationship between the debt coverage ratio and the cap rate, which supports my theory. This relationship is economically significant. A 100 basis point increase in the cap rate corresponds to a 571 basis point decrease in the debt coverage ratio for apartment, a 901 basis point decrease for office, a 1,135 basis point decrease for industrial, and a 926 basis point decrease for retail. Figure 11 plots the residuals from the time series regressions that include both Tightening Standard and the cap rate, which are stationary according to unit root tests.

Overall, results in Tables 4 to 7 generally, but not always, support the theoretical predictions of my model. When the results are not consistent with my predictions, they are typically mixed and weak. The strongest and probably the most surprising result is that financial leverage is actually counter-cyclical for commercial real estate, which is predicted by my model. The loan to value ratio tends to be high when property values are low (high cap rates). This finding suggests that there is endogenous correlation between financial leverage and property values, which should not be interpreted as causation. Further, the possible impact of exogenous regulations imposed by the government needs to be analyzed with the endogenous relationships between financial leverage and property values taken into account. For example, a strict loan to value ratio requirement during market booms may be irrelevant if the equilibrium loan to value ratio tends to be low.

VI. Conclusions

This paper develops a simple theoretical model, in which differences in property owners and mortgage lenders' discount rates jointly determine equilibrium mortgage interest rates, financial leverage, and property values. My model provides some original and testable predictions. The most surprising prediction is that financial leverage is counter-cyclical for commercial real estate: the loan to value ratio tends to be high for newly originated mortgages when property cap rates are high. Despite being counter-intuitive, co-integration analysis on data from the ACLI database and the Federal Reserve Senior Loan Officer Survey provides strong support for this prediction. To my knowledge, both the theoretical prediction and the strong empirical evidence on the counter-cyclical financial leverage are original and seem to be an important contribution to the literature. Further, these findings suggest the importance of taking into account the endogeneity of financial leverage when policy makers try to use it as a new "tool" to affect the financial market and the economy.

Appendix A

Below I present the partial derivatives of the loan interest rate, the loan to value ratio, the debt coverage ratio, and the cap rate with respect to the intercept terms of property owners and lenders' discount rate functions.

$$\frac{dr^*}{da_O} = \frac{b_L}{b_L - b_O} > 0$$

$$\frac{dr^*}{da_L} = \frac{-b_O}{b_L - b_O} < 0$$

$$\frac{dLTV^*}{da_O} = \frac{a_O\left(a_O b_L - a_L b_O\right) + a_L b_O^2 + a_L b_O b_L}{\left(a_O b_L - a_L b_O\right)^2} > 0$$

$$\frac{dLTV^*}{da_L} = \frac{\left(b_O - b_L\right)\left(a_O^2 + a_O b_O\right)}{\left(a_O b_L - a_L b_O\right)^2} < 0$$

$$\frac{dDCR^*}{da_O} = \frac{b_O - b_L}{\left(a_O - a_L\right)^2} < 0$$

$$\frac{dDCR^*}{da_L} = \frac{b_L - b_O}{\left(a_O - a_L\right)^2} > 0$$

$$\frac{dcaprate^*}{da_O} = 1 > 0$$

$$\frac{dcaprate^*}{da_L} = 0$$

References

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Table 1. Model Comparative Statics

This table reports signs of partial derivatives of the equilibrium loan interest rate, loan to value ratio, debt coverage ratio, and cap rate with respect to the intercept terms of property owners and lenders' discount rate functions.

	$u = a_O$	$u = a_L$
$\frac{dr^*}{du}$	>0	<0
$\frac{dLTV^*}{du}$	>0	<0
$\frac{dDCR^*}{du}$	<0	>0
$\frac{dcaprate^*}{du}$	>0 (=1)	=0

Table 2. Data Summary

This table reports summary statistics for the mortgage interest rate, loan to value ratio, debt coverage ratio, and cap rate of apartment, office, retail, and industrial properties from the ACLI database from 1990:2 to 2011:2. It also reports the same statistics for Tightening Standard in the same period, which is the "net percentage of domestic respondents tightening standards for commercial real estate loans" reported in the Federal Senior Loan Officer Opinion Survey.

Variable	Mean	Std. Dev.	Min	Median	Max
Interest rate					
Apartment	7.01%	1.40%	4.42%	7.02%	9.92%
Office	7.17%	1.43%	4.80%	7.19%	9.91%
Retail	7.19%	1.35%	4.78%	7.29%	9.91%
Industrial	7.11%	1.36%	4.67%	7.21%	9.89%
Loan to value ratio					
Apartment	68%	4%	54%	68%	77%
Office	67%	6%	49%	67%	89%
Retail	66%	4%	54%	67%	72%
Industrial	69%	4%	57%	69%	79%
Debt coverage ratio					
Apartment	1.55	0.21	1.22	1.49	2.19
Office	1.66	0.26	1.18	1.61	2.97
Retail	1.58	0.20	1.20	1.55	2.26
Industrial	1.57	0.20	1.30	1.55	2.17
Cap rate					
Apartment	7.98%	1.15%	5.80%	8.50%	9.60%
Office	8.73%	1.25%	6.10%	9.10%	11.10%
Retail	8.54%	0.93%	6.60%	8.90%	10.20%
Industrial	8.91%	0.98%	6.80%	9.15%	10.50%
Tightening standard					
	17%	26%	-24%	10%	87%

Table 3. Correlations

This table reports correlations between the mortgage interest rate, loan to value ratio (LTV), debt coverage ratio (DCR), cap rate, and Tightening Standard (Tightening) for apartment, office, retail, and industrial properties.

	Apartment			
	LTV	DCR	Cap rate	Tightening
Interest rate	0.51	-0.66	0.78	0.07
LTV		-0.45	0.58	-0.25
DCR			-0.27	-0.19
Cap rate				-0.19
	Office			
	LTV	DCR	Cap rate	Tightening
Interest rate	0.57	-0.67	0.76	0.10
LTV		-0.59	0.69	-0.33
DCR			-0.39	-0.08
Cap rate				-0.22
	Retail			
	LTV	DCR	Cap rate	Tightening
Interest rate	0.50	-0.79	0.65	0.12
LTV		-0.59	0.61	-0.28
DCR			-0.42	-0.03
Cap rate				-0.19
	Industrial			
	LTV	DCR	Cap rate	Tightening
Interest rate	0.40	-0.83	0.74	0.12
LTV		-0.56	0.53	-0.41
DCR			-0.50	-0.07
Cap rate				-0.30

Table 4. Comparative Statics of Mortgage Interest Rates

This table reports regressions of quarterly series of the mortgage interest rate against the time series of Tightening Standard, the cap rate, and both for apartment, office, industrial, and retail properties respectively. ** indicates significance at the 1% level, and * indicates significance at the 5%. If the existence of unit roots is rejected with the augmented Dickey-Fuller test, the value for "co-integration" is "Yes"; otherwise, it is "No".

	Apartment			Office			Industrial			Retail		
Tightening Standard	0.00		0.01**	0.01		0.02**	0.01		0.02**	0.01		0.01**
	(0.01)		(0.00)	(0.01)		(0.00)	(0.01)		(0.00)	(0.01)		(0.00)
Cap rate		0.95**	1.00**		0.86**	0.93**		1.02**	1.17**		0.95**	1.02**
		(0.08)	(0.08)		(0.08)	(0.08)		(0.10)	(0.09)		(0.12)	(0.11)
Adjusted R2	-0.01	0.61	0.65	-0.00	0.57	0.64	0.00	0.54	0.66	0.00	0.42	0.48
Co-integration	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes

Table 5. Comparative Statics of Mortgage Interest Rate Risk Premium

This table reports regressions of quarterly series of the mortgage interest rate risk premium (mortgage interest rate minus the 10-year Treasury yield) against the time series of Tightening Standard, the cap rate risk premium (cap rate minus the 10-year Treasury yield), and both for apartment, office, industrial, and retail properties respectively. ** indicates significance at the 1% level, and * indicates significance at the 5%. If the existence of unit roots is rejected with the augmented Dickey-Fuller test, the value for "co-integration" is "Yes"; otherwise, it is "No".

	Apartment			Office			Industrial			Retail		
Tightening Standard	0.01**		0.01**	0.02**		0.02**	0.02**		0.02**	0.02**		0.02**
	(0.00)		(0.00)	(0.01)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)
Cap rate		0.37**	0.35**		0.37**	0.37**		0.40**	0.41**		0.38**	0.35**
		(0.06)	(0.05)		(0.07)	(0.05)		(0.07)	(0.05)		(0.06)	(0.05)
Adjusted R2	0.27	0.29	0.54	0.27	0.25	0.53	0.29	0.27	0.58	0.26	0.29	0.52
Co-integration	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 6. Comparative Statics of Loan to Value Ratios

This table reports regressions of quarterly series of the mortgage loan to value ratio against the time series of Tightening Standard, the cap rate, and both for apartment, office, industrial, and retail properties respectively. ** indicates significance at the 1% level, and * indicates significance at the 5%. If the existence of unit roots is rejected with the augmented Dickey-Fuller test, the value for "co-integration" is "Yes"; otherwise, it is "No".

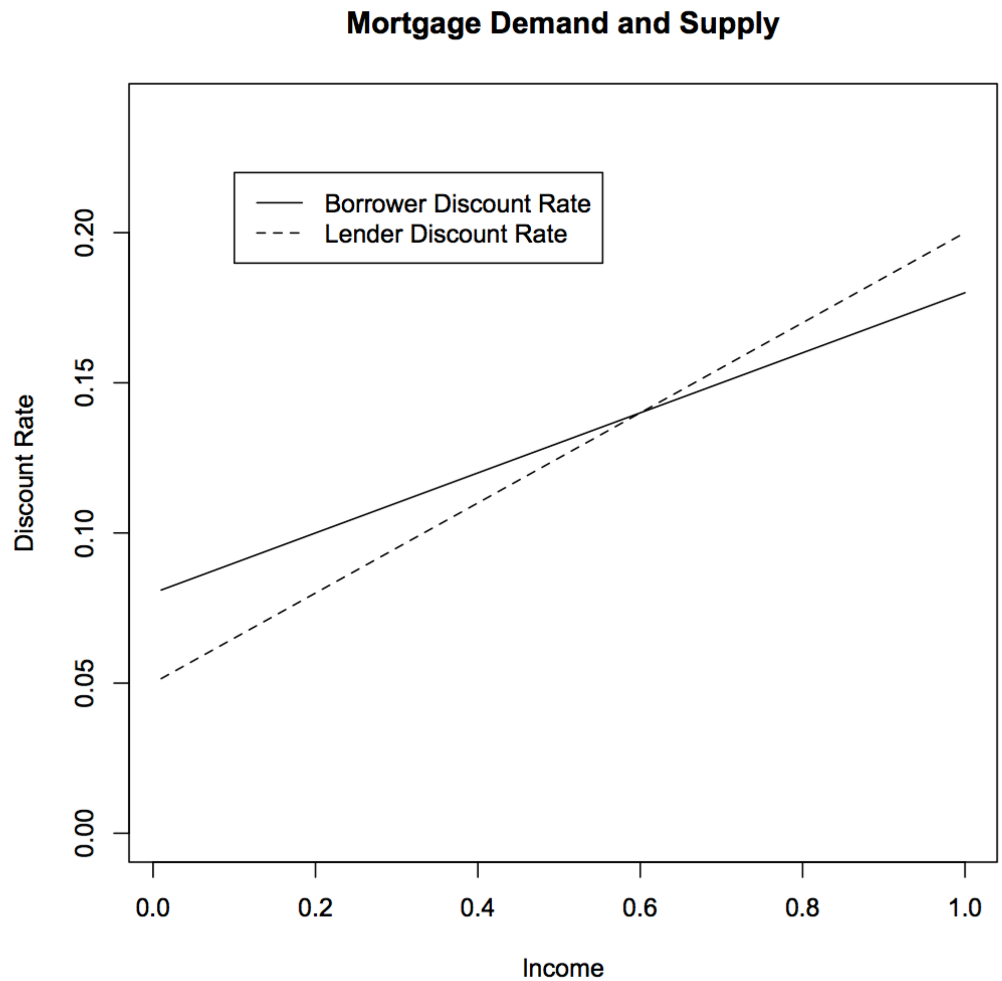
	Apartment			Office			Industrial			Retail		
Tightening Standard	-0.04*		-0.02	-0.08**		-0.05**	-0.06*		-0.04**	-0.04		-0.03*
	(0.02)		(0.01)	(0.03)		(0.02)	(0.02)		(0.01)	(0.02)		(0.01)
Cap rate		1.94**	1.86**		3.55**	3.34**		2.23**	1.89**		2.68**	2.54**
		(0.30)	(0.03)		(0.42)	(0.41)		(0.39)	(0.39)		(0.38)	(0.38)
Adjusted R2	0.05	0.33	0.34	0.10	0.47	0.49	0.16	0.27	0.34	0.07	0.37	0.39
Co-integration (Reject unit root)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 7. Comparative Statics of Debt Coverage Ratios

This table reports regressions of quarterly series of the mortgage debt coverage ratio against the time series of Tightening Standard, the cap rate, and both for apartment, office, industrial, and retail properties respectively. ** indicates significance at the 1% level, and * indicates significance at the 5%. If the existence of unit roots is rejected with the augmented Dickey-Fuller test, the value for "co-integration" is "Yes"; otherwise, it is "No".

	Apartment			Office			Industrial			Retail		
Tightening Standard	-0.15		-0.20*	-0.08		-0.17	-0.05		-0.18*	-0.02		-0.08
	(0.09)		(0.08)	(0.11)		(0.10)	(0.08)		(0.07)	(0.08)		(0.08)
Cap rate		-4.84*	-5.71**		-8.24**	-9.01**		-9.93**	-11.35**		-8.82**	-9.26**
		(1.92)	(1.90)		(2.14)	(2.17)		(1.91)	(1.94)		(2.11)	(2.14)
Adjusted R2	0.02	0.11	0.34	-0.01	0.14	0.16	-0.00	0.24	0.28	-0.01	0.17	0.17
Co-integration (Reject unit root)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Figure 1.



Mortgage Demand and Supply
Borrower Discount Rate
Lender Discount Rate
Discount Rate
0.00
0.05
0.10
0.15
0.20
0.0
0.2
0.4
0.6
0.8
1.0
Income

Figure 2.

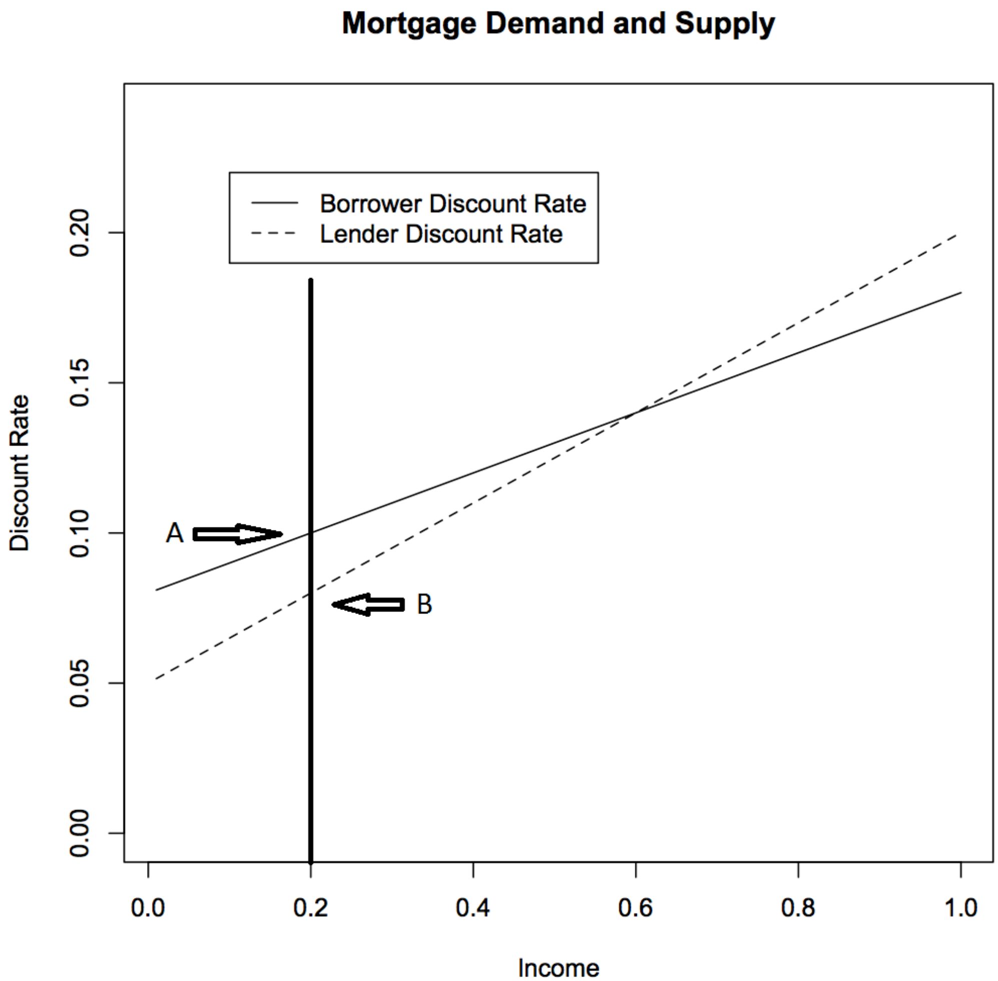
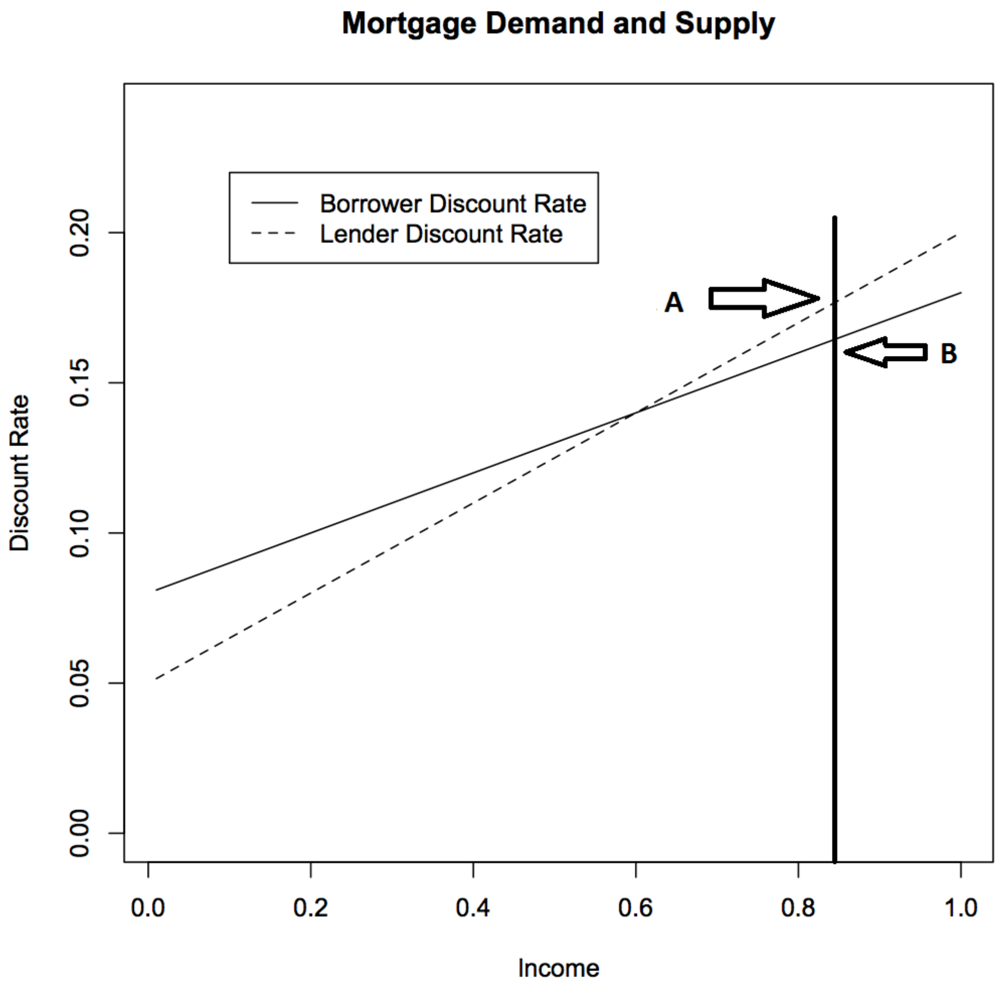

Mortgage Demand and Supply
Borrower Discount Rate
Lender Discount Rate
A
B
Discount Rate
0.00
0.05
0.10
0.15
0.20
0.0
0.2
0.4
0.6
0.8
1.0
Income

Figure 3.



Mortgage Demand and Supply
Borrower Discount Rate
Lender Discount Rate
A
B
Discount Rate
0.00
0.05
0.10
0.15
0.20
0.0
0.2
0.4
0.6
0.8
1.0
Income

Figure 4.

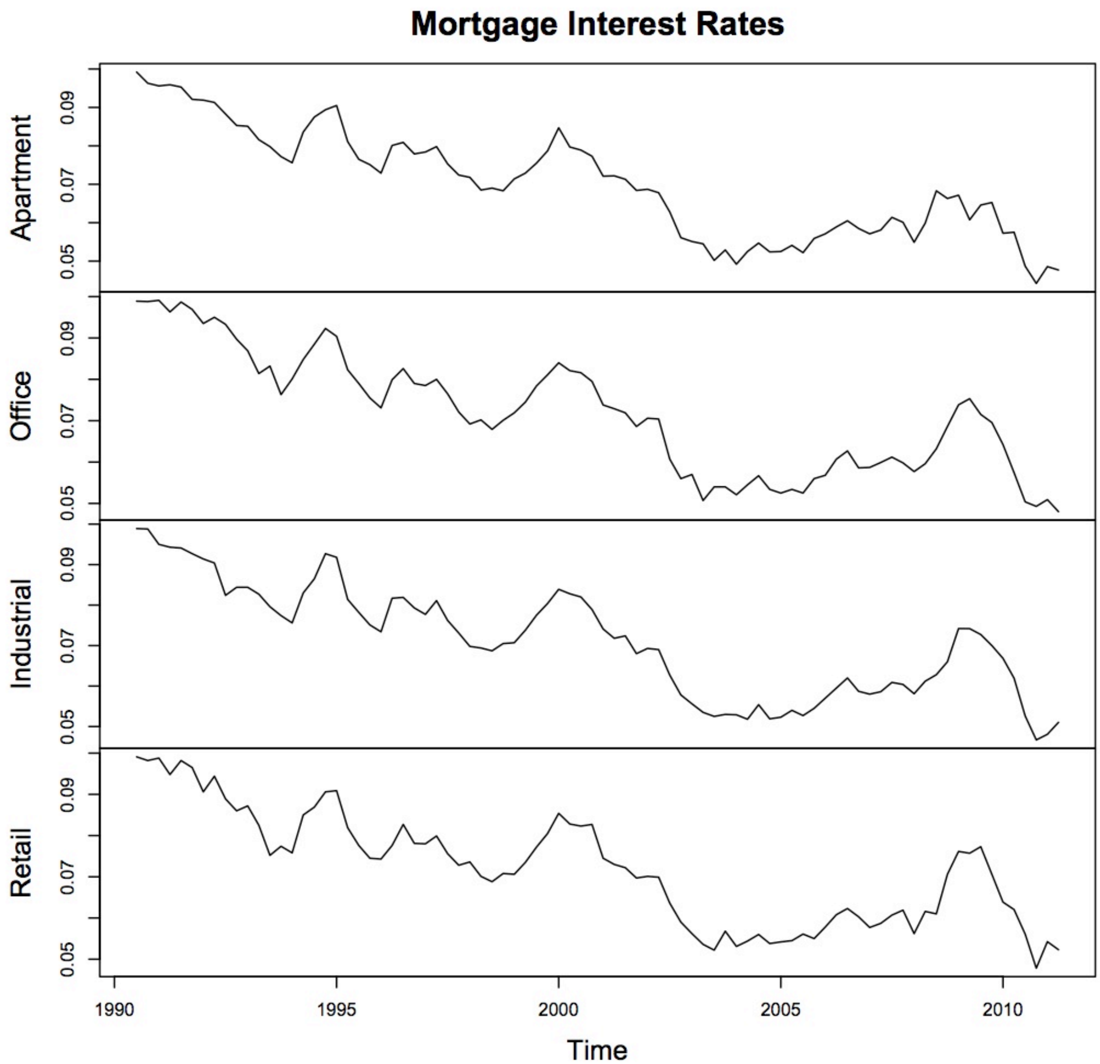
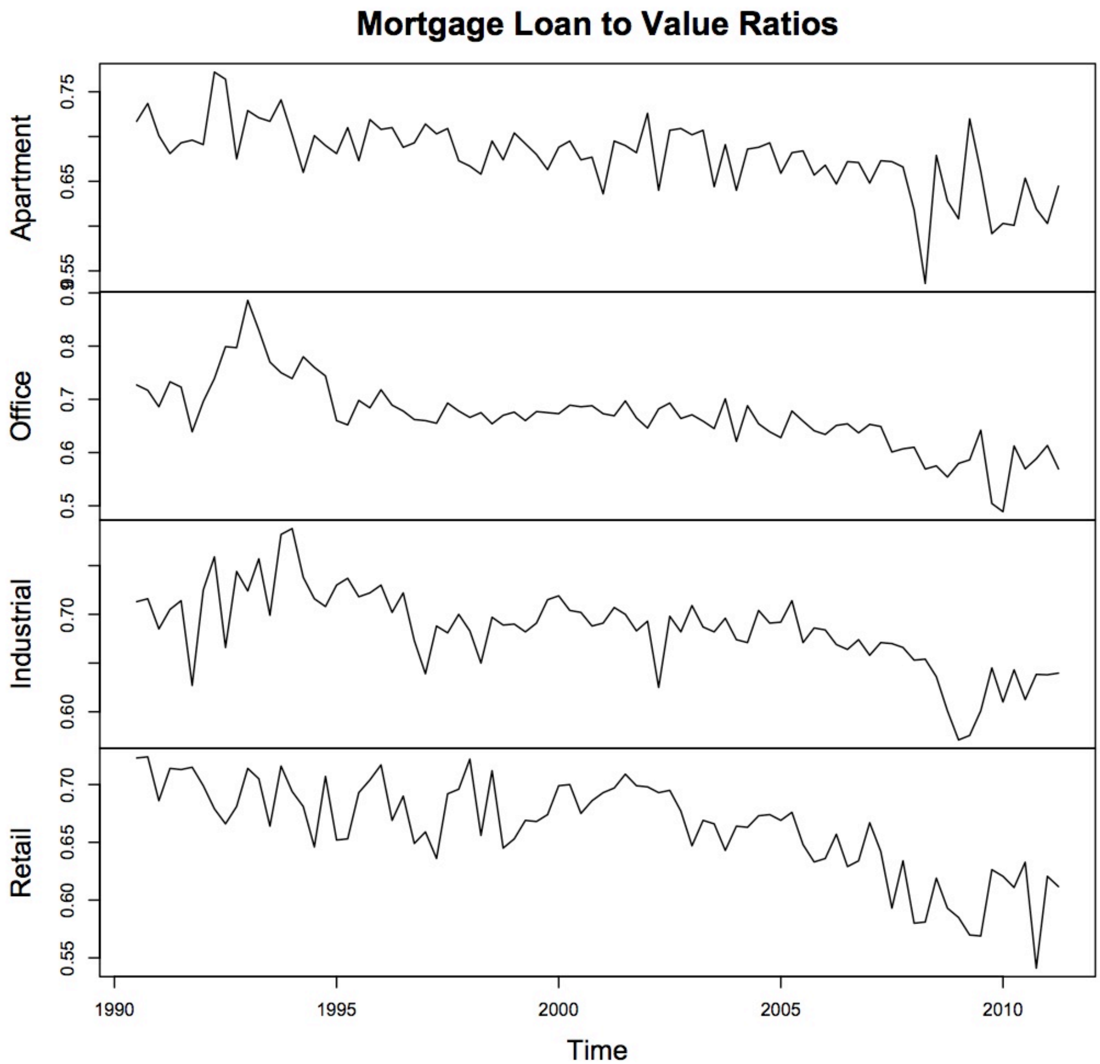
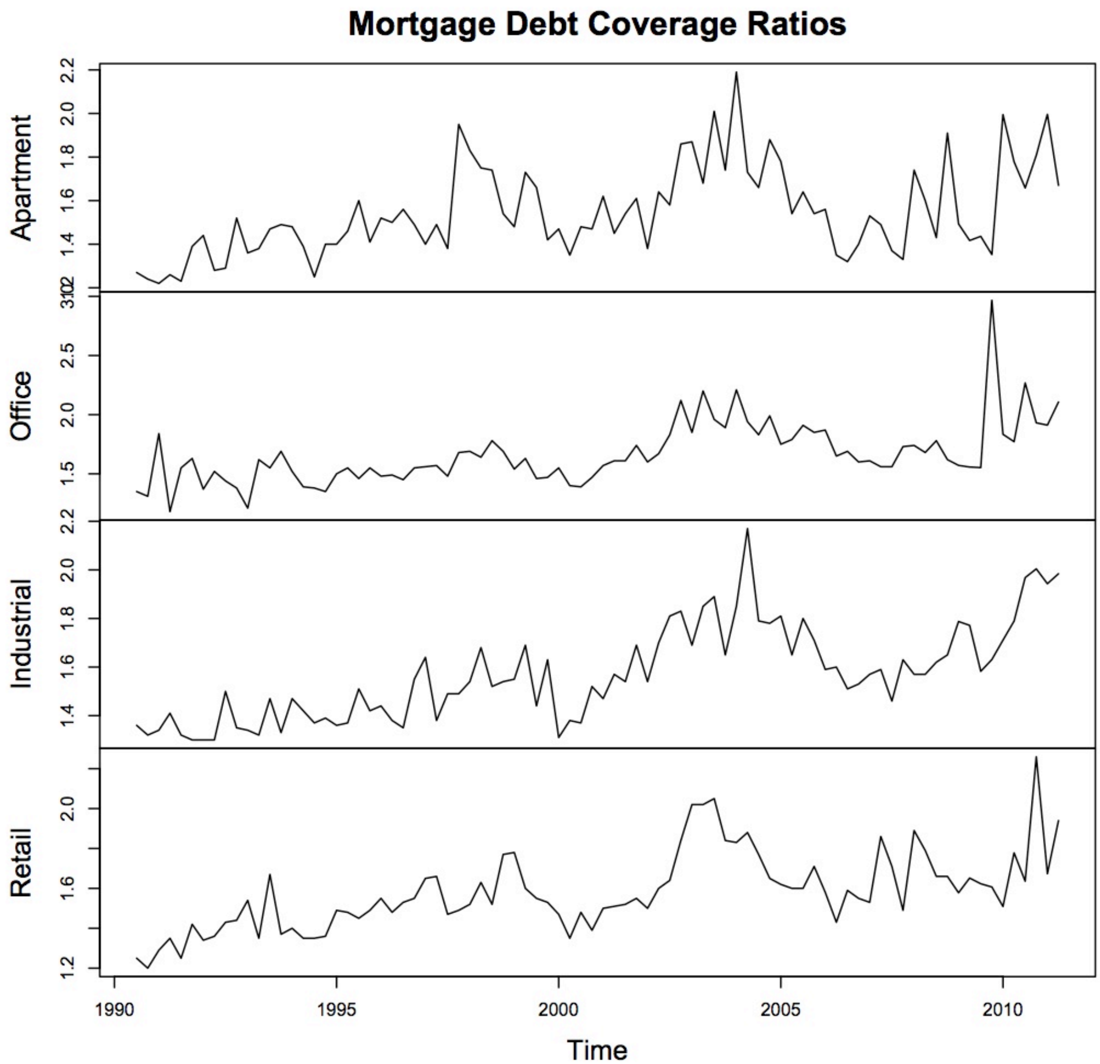

Mortgage Interest Rates
Apartment
Office
Industrial
Retail
0.05
0.07
0.09
1990
1995
2000
2005
2010
Time

Figure 5.



Mortgage Loan to Value Ratios
Apartment
Office
Industrial
Retail
0.75
0.65
0.55
0.9
0.8
0.7
0.6
0.5
0.70
0.60
0.70
0.65
0.60
0.55
1990
1995
2000
2005
2010
Time

Figure 6.



Mortgage Debt Coverage Ratios
Apartment
Office
Industrial
Retail
Time
1990
1995
2000
2005
2010

Figure 7.

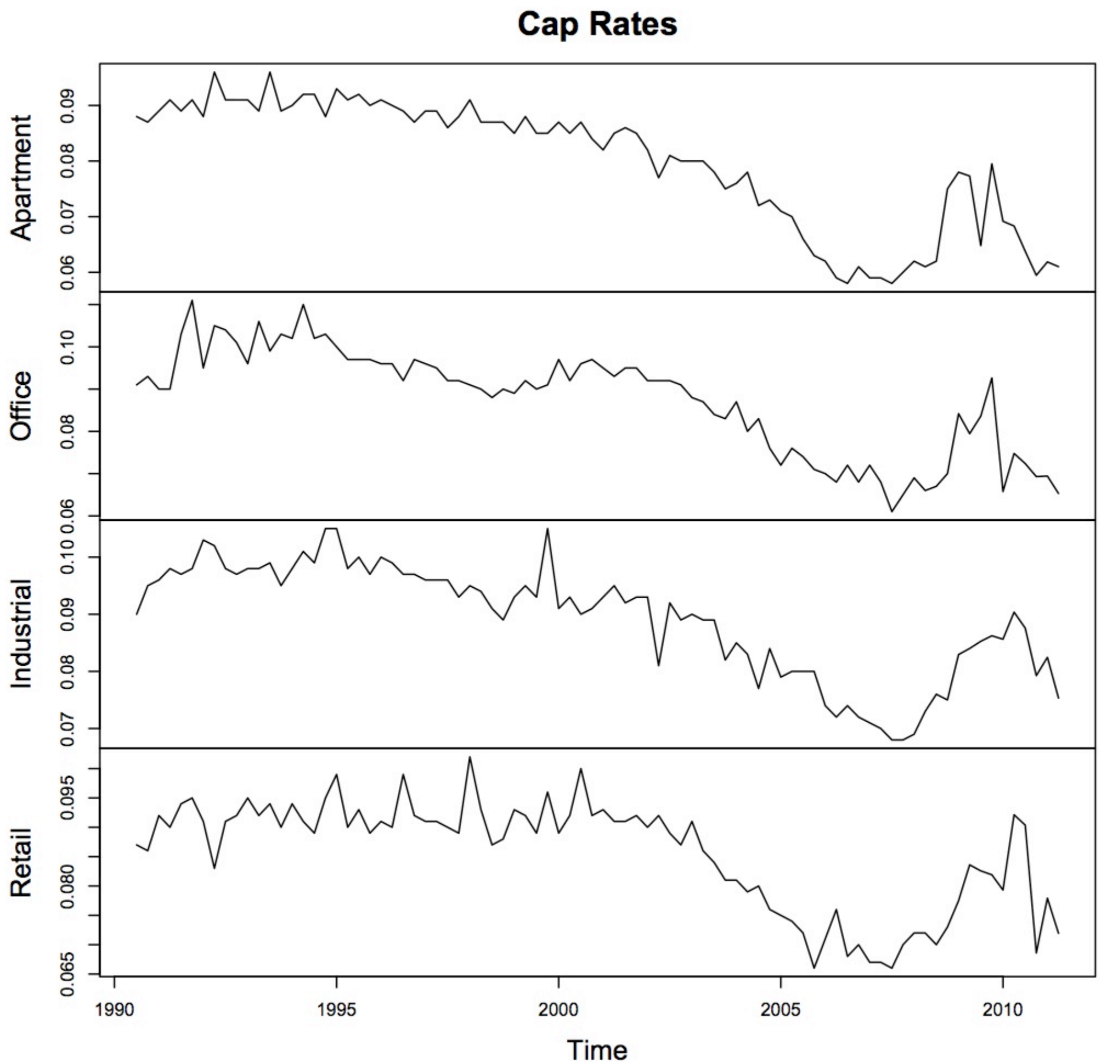

Cap Rates
Apartment
Office
Industrial
Retail
0.06
0.07
0.08
0.09
0.06
0.08
0.10
0.07
0.08
0.09
0.10
0.065
0.080
0.095
1990
1995
2000
2005
2010
Time

Figure 8.

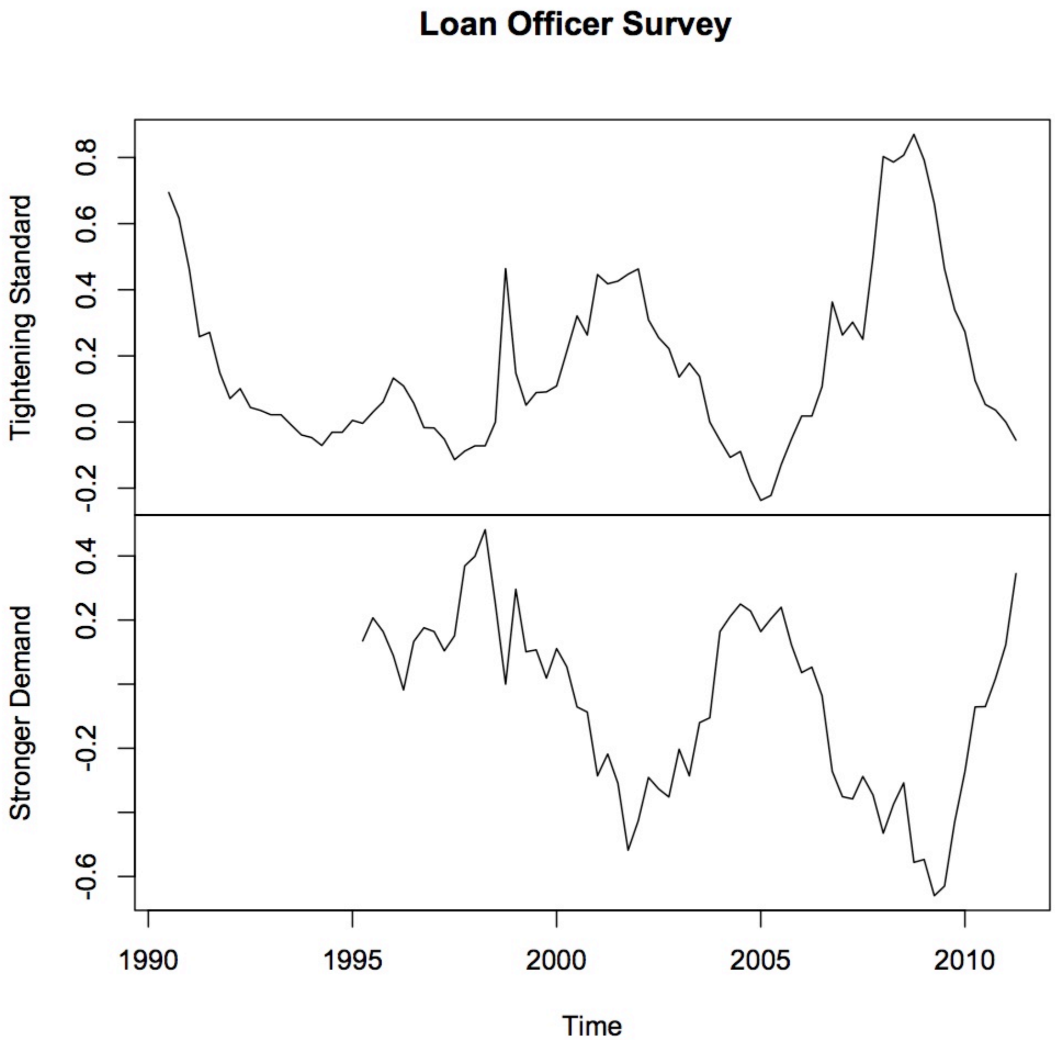

Loan Officer Survey
Tightening Standard
0.8
0.6
0.4
0.2
0.0
-0.2
Stronger Demand
0.4
0.2
-0.2
-0.6
1990
1995
2000
2005
2010
Time

Figure 9.


Mortgage Interest Rate Residuals
Apartment
Office
Industrial
Retail
Time
1990
1995
2000
2005
2010

Figure 10.

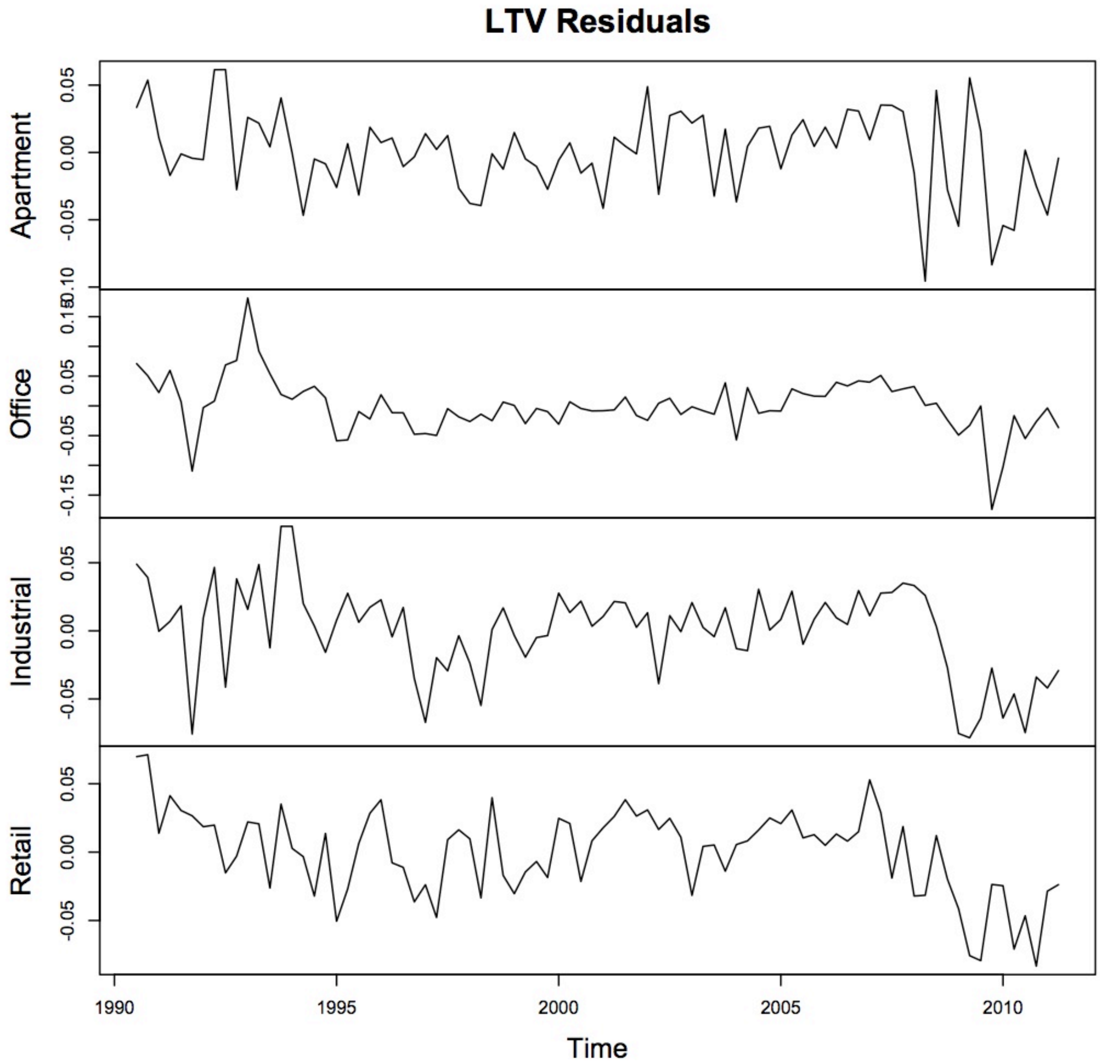
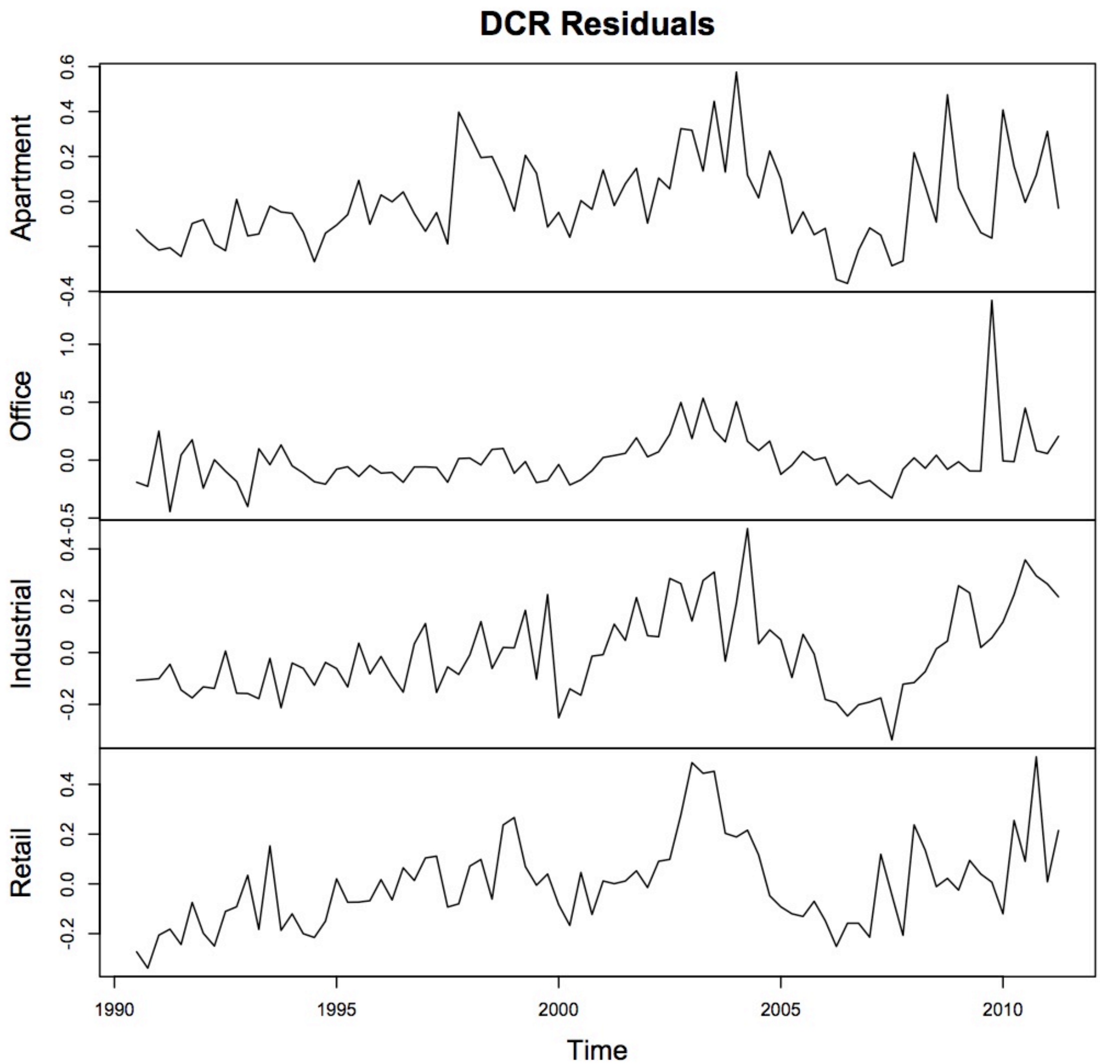

LTV Residuals
Apartment
Office
Industrial
Retail
0.05
0.00
-0.05
-0.10
0.15
0.05
-0.05
-0.15
0.05
0.00
-0.05
0.05
0.00
-0.05
1990
1995
2000
2005
2010
Time

Figure 11.


DCR Residuals
Apartment
Office
Industrial
Retail
0.6
0.4
0.2
0.0
-0.4
1.0
0.5
0.0
-0.5
0.4
0.2
0.0
-0.2
0.4
0.2
0.0
-0.2
1990
1995
2000
2005
2010
Time