The Returns of Private and Public Real Estate*

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Abstract

We propose a general equilibrium model in which differences between the cost of capital of private and public real estate may yield an explanation for how their returns are integrated.

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1 Introduction

Institutional investors hold real estate directly as well as by holding public securities like REITs. The advantages of public ownership are generally described as coming from the transparency and liquidity of REITs. However, these advantages do not appeal to all investors. Indeed, a number of institutions prefer the flexibility associated with how they mark to market their private real estate as well as the control associated with direct ownership.

The investors that prefer the private and public ownership alternatives are often distinct, and the amount of capital that is available to these segments can shift over time. One possible cause for such a shift, which is consistent with anecdotal evidence, is shocks to the supply of debt capital that is available to the private investors. For example, in the early 1990s, the availability of bank debt for private real estate investments was substantially reduced, raising the relative cost of capital in the private markets relative to the public markets. Later, in the mid-2000s, the supply of debt for real estate increased, reducing the relative cost of capital in the private markets. The emergence of REITs in the early to mid-1990s and the conversions of REITs from public to private ownership in the 2005 to 2007 period is consistent with the idea that changes in the relative cost of capital in these partially segmented markets is an important determinant of ownership structure in this industry. Changing sentiment in the equity markets can also affect the relative supply of capital in private and public markets. For example, flows into small cap value mutual funds, or more importantly REIT mutual funds, influence the market prices of REITs relative to the value of the real estate they hold.

This paper examines how shifts in the supply of capital in these segmented markets affect the prices of public real estate securities relative to prices of the actual real estate. To do this we develop a model where the value of real estate, as well as the price of REITs, fluctuates because of changing discount rates in the two markets, as well as because of changes in the cash flows generated by real estate. While the values in these markets are separately determined as functions of their independent discount rates, the private and public prices are linked by the fact that real estate flows from private to public ownership when price differences in these markets are sufficiently high.

The analysis in this paper should be of particular interest to private investors, like pension funds and endowments, which are required to report the fair values of their investments under FASB 157. The question that arises in this context is the extent to which prices observed in the more liquid public markets can be used as comps to mark to market privately held real estate. A related question is the extent to which the time series of public market prices can be used to gauge the risks of the private investments. For example, can an endowment fund use the observed correlations between REIT returns and stock market returns to estimate the correlation between their equity portfolio and their real estate portfolio?

The traditional view of financial economists has been that prices in the more liquid and efficient public markets provide the best comps for pricing privately held real estate. However, as we illustrate in this paper, if the public and private markets are partially segmented, there can be substantial differences between the market values of REITs and the value of the real estate they hold. In addition, the risk characteristics of REITs and real estate differ, and as we show, the riskiness of these investments depend on the extent to which REITs are selling for a discount or premium relative to the real estate assets they hold. For these reasons, it is not appropriate to simply use public market prices as proxies for value in private markets.

Of course, appraisal-based measures of real estate values have other problems. Since appraisals tend to be lagged and smoothed estimates of the true underlying values, marking-to-market with purely appraisal-based values produce understated (overstated) asset values in rising (falling) markets. Hence, as we show, real estate portfolios can be more accurately marked to market with a combination of appraisal-based prices and REIT prices. In addition, the weight that one would place on REIT versus appraisal prices will change over time and depend on the estimated discount rate or premium of REIT prices relative to the value of the underlying real estate.

We are, of course, not the first to study the relation between private real estate prices and REIT prices. Existing papers document the correlation between real estate returns and REIT returns[1] as well as the correlation between REIT returns and various stock return indexes.[2] In addition,

[1] See, among others, Barkham and Geltner (1995); Pagliari, Scherer, and Mononoli (2005); and Riddiough, Moriarty, and Yeatman (2005).

[2] See among others Gyourko and Keim (1992); Oppenheimer and Grissom (1998); Ling and Naranjo (1999); Glascock, Lu, and So (2000); and Chiang, Lee, and Wisen (2005).

we are not the first to note that REIT returns may be influenced by the returns of non-real estate stock returns.[3] However, we are the first to develop a theoretical model that describes how these relationships can be influenced by flows into and out of the public and private markets and how the correlations between public and private market prices depends on the magnitudes of the discounts and premiums of REITs.[4]

The paper proceeds as follows: Section 2 develops a static equilibrium model for REIT prices. Section 3 presents the dynamic, continuous-time extension as well as its empirical implications. Section 4 shows how values of real assets can be inferred by observing the prices of public assets as well as delayed and smoothed appraisal-based prices for the private assets. Section 5 shows how we calibrate our model and tests its empirical implications, and the last Section presents our conclusions.

2 A Static Equilibrium Model of REIT Prices

As a first step, we develop a model of REIT prices in a static setting. This allows us to highlight the economic forces that drive the equilibrium in our fully dynamic model, which is developed in the next section.

The model assumes that real estate generates cash flows at a rate of \$ L per year, which we initially assume to be certain. The model also assumes two distinct types of investors, which we will refer to as individuals and institutions. Institutions purchase real estate outright and hold them privately while still maintaining a well-diversified portfolio. The opportunity cost of capital for real estate held in this form is assumed to be r^* (the "private discount rate"), which implies a private valuation of $P^* = L/r^*$. In contrast, individual investors are unable to directly hold real estate, and instead hold real estate indirectly by purchasing shares of REITs, which directly purchase the real estate. The price P of REITs as well as the associated public discount rate r, is stochastic and is determined within our equilibrium model.

We assume that the demand for REITs by individuals is downward sloping, since their optimal exposures to REITs depends on the expected return premium associated with this asset class. This

[3] See for example N. and Webb (1992); Giliberto and Mengden (1996); and Clayton and MacKinnon (2003).

[4] Premiums and discounts arise in our model for reasons that are related to those modeled in Cherkes, Sagi, and Stanton (2010). Our work extends their model of the closed end fund discount and addresses issues that are unique to the setting of real estate assets.

demand for REITs shifts over time, reflecting what we will refer to as market conditions or investor sentiment. For example, investors will want to supply either more or less capital to this sector depending on the expected returns and risks of alternative public investment opportunities.

The number of real estate units held by REITs is endogenous, and these units can be bought and sold by REITs at the private market price with a transaction cost $C_{PR} = c_{PR}L$. For simplicity, we have made the price of real estate essentially exogenous in this setting, by assuming that the supply of capital to private real estate is inelastic at the exogenously specified required rate of return on privately held real estate. What this means is that the movement of real estate units to and away from REITs has no effect on the required rate of return in the private real estate market. However, given the elasticity of the supply of capital to REITs, assets moving into and out of REITs do influence discount rates in the public markets.

Figure 1 graphically depicts the supply and demand curves for assets held as REITs. In the figure, the value K_t represents real estate holdings in the REIT sector due to prior-period decisions. Additional supply of properties by institutions ensures that the equilibrium price of REITs is bounded below by the private market value less transaction costs associated with transferring assets from REIT to private holders: $P \geq l/r^* - c_{RP}L$. This limits the negative impact that a low level of individual demand can have on REIT prices. Conversely, due to demand by institutions, the equilibrium price of REITs is bounded above by the private market value plus transaction costs from private to REIT ($P \leq L/r^* + c_{PR}L$) and this limits the positive impact that high individual demand can have on REIT prices.

Some intuition for REIT return dynamics can be developed by considering comparative statics with respect to demand from individuals. At moderate values, changes in individual demand directly translate into REIT price shocks, since there will be no response from the private sector. At high or low levels of individual demand, however, purchases from or sales to the private sector will attenuate the price impact of these demand shocks. As a consequence, the volatility of REIT returns will vary depending on the level of demand relative to holdings of property in the REIT sector. Similarly, REIT and private returns should be more highly correlated in situations in which supply responses are expected to occur.

In the next section we extend this simple model and develop a fully dynamic, continuous time model that we can calibrate and take to the data.

3 A Dynamic Equilibrium Model of REIT Returns

This section develops a dynamic model of an economy that consists of N units of real estate that can be held either directly or through a publicly traded firm. Within the context of this model we will value a representative firm that holds one unit of real estate that produces a flow of $\$L_t$ per year. We will start by presenting the elements of the model, and will then characterize the equilibrium.

3.1 The Economy

The elements of the economy are as follows:

- A unit of property produces a flow of $L_t dt$ during a small time period of dt. We assume that the flow is log-normal:

$$dL_t/L_t = \mu_L dt + \sigma_L dW_t^L, \tag{1}$$

- The market for privately-held real estate is perfectly contestable. In particular, an infinite amount of capital is available to enter (exit) the private sector if the price of assets producing cash flows L_t is lower (higher) than the discounted value of the cash flows using an exogenous opportunity cost of capital r_t^*.

- The opportunity cost of private capital is a constant $r_t^* = r^*$.

- The public discount rate r_t is stochastic, and can be described by the general functional form

$$dr_t = \mu_r(r_t)dt + \sigma_r(r_t)dW_t^r + dD_t - dU_t, \tag{2}$$

  D and U are strictly increasing stochastic processes that describe the effects of REIT supply on the public discount rate. Specifically, when REITs sell units of real estate to private buyers, dU is positive and the public discount rate decreases, and when REITs acquire real estate from private buyers, dD is positive and the public discount rate increases. When the properties held by REITs remains unchanged the instantaneous public discount rate is a diffusion. In our numerical solutions, we assume that the exogenous shocks to r_t follow a Cox-Ingersoll-Ross process:

$$dr_t = k_r(\tilde{r} - r_t)dt + \sigma_r \sqrt{r_t} dW_t^r + dD_t - dU_t, \tag{3}$$

where W^r is a standard Brownian Motion. This diffusion structure ensures that the public discount rate remains positive.

- The Brownian motions driving r and l are correlated with a correlation coefficient ρ, that is, $dW_t^L \times dW_t^r = \rho dt$.
- Any asset can be either privately or publicly held, with the possible ownership states for an asset represented by the set $\mathbb{I} = \{P, R\}$, where state $i = R$ corresponds to the public (REIT) state and state $i = P$ corresponds to the private state.
- When privately held assets are sold to REITs a transaction cost of $c_{PR}L_t$ applies, and when REIT assets are sold to private investors a cost of $c_{RP}L_t$ applies. All transaction costs are paid by the REIT.
- No transaction costs are incurred when privately held assets are sold to other private investors.

3.2 The Value Function

The value of an asset in state i is denoted by V^i, where $i = P$ represents the privately held asset while $i = R$ represents the value of a REIT. A *strategy* is a rule π^i by which an asset switches between the states P (i.e., being privately owned) and R (i.e., being part of a REIT). Starting with an initial state i, a strategy is the sequence of stopping times $\tau_0, \tau_1, ...$ such at at any time τ_n the asset switches its type from state $k_{n-1} \in \{P, R\}$ into state $k_n \in \{P, R\}$. For strategy π^i the property begins in state i so $\tau_0 = 0$ and $k_0 = i$. For each time t we denote the current state of the company by $I_t \in \{P, R\}$. Each time t when the company is in state I_t the relevant discount rate on the interval $[t, t + dt)$ is:

$$R_t(I_t) = \begin{cases} r_t, & \text{if } I_t = R \\ r^*, & \text{if } I_t = P \end{cases} \tag{4}$$

The value of an asset is the sum of discounted cash flows until the first time the asset switches its type (to type $j \neq i$), minus the transaction costs necessary to switch type, plus the discounted value of that asset after the switch. Denoting by θ the first time where a switch occurs for an asset of type i, the value V^i of that asset will be:

$$\begin{aligned} & V^i(r, L) \\ & = \sup \mathbb{E}\left[\int_0^\theta e^{-\int_0^t R_s(I_s)ds} L_t dt - e^{-\int_0^\theta R_s(I_s)ds} c_{i,\ j} L_\theta + e^{-\int_0^\theta R_s(I_s)ds} V^{I_\theta}(r_\theta, L_\theta) \middle| r_0 = r,\ L_0 = L\right]. \end{aligned} \tag{5}$$

Before considering equilibrium behavior, we clarify how the value functions V^i, which represent the discounted value of cash flows produced by assets currently held in private ($i = P$) or public ($i = R$) form, differ from the observed prices paid for real estate assets when they change hands. This distinction depends on both the level of transaction costs as well as which party, purchaser or seller, incurs the costs. We take the view that it is costly for REITs to access capital markets when transferring funds to or from investors, for example due to the cost of issuing trust units. Under this assumption, the real estate price will represent the proceeds transferred from/to the private buyer/seller. In equilibrium, therefore, all real estate transaction prices will reflect private valuations and REITs will trade at a premium or discount to these values. Under our assumption of zero transaction costs for asset sales between private parties, the observed real estate prices associated with such transactions will be $V^P(r, L)$ in equilibrium. Thus, regardless of the parties involved in a real estate transaction, prices in our model will always reflect private valuations.

3.3 The Equilibrium

An equilibrium in our model consists of return processes for publicly held real estate, r_t, and privately held real estate, r^*, and associated capital flows to and from the private sector. We do not explicitly model the capital flows and instead characterize the general form of the flows and their impact on REIT prices and discount rates in the following Theorem.

Theorem 1 *Let V^P the value of the privately held asset and V^R that of the publicly held asset. Then the value function V^P and V^R satisfy:*

$$
\begin{aligned}
V^R(r, L) &= L\hat{\phi}_R(r), \\
V^P(r, L) &= L/(r^* - \mu_L).
\end{aligned}
\tag{6}
$$

where the function $\hat{\phi}^R$ *satisfies the following:*

1. *There exists two constants* $0 \leq \underline{r} < \overline{r} \leq \infty$ *such that if* $r \in (\underline{r}, \overline{r})$ *the function* $\hat{\phi}_R$ *satisfies:*

$$
\frac{\sigma_r(r)^2}{2}\hat{\phi}_R''(r) + [\mu_r(r) + \rho\sigma_L\sigma_r(r)]\hat{\phi}_R'(r) - r\hat{\phi}_R(r) + 1 = 0 \tag{7}
$$

2. *At* $r = \overline{r}$ *institutions purchase public assets and hold them privately. The function* $\hat{\phi}_R$ *satisfies:*

$$
\begin{aligned}
\hat{\phi}_R(\overline{r}) &= 1/(r^* - \mu_L) - c_{RP} \\
\hat{\phi}_R'(\overline{r}) &= 0.
\end{aligned}
\tag{8}
$$

3. *If at* $r = \underline{r}$ *the function* $\hat{\phi}_R$ *satisfies the following conditions:*

$$
\begin{aligned}
1/(r^* - \mu_L) &= \hat{\phi}_R(\underline{r}) - c_{PR} \\
\hat{\phi}_R'(\underline{r}) &= 0,
\end{aligned}
\tag{9}
$$

   then REITs purchase privately owned assets. If those conditions are not met, no privately held assets become public.

4. *Supply of private capital to the REIT sector at* $r_t = \underline{r}$ *(if conversions occur) and the demand for assets held by REITs at* $r_t = \overline{r}$ *maintain the public discount rate* r_t *in the interval* $[\underline{r}, \overline{r}]$*. That is, there exist two stochastic processes* D *and* U *such that:*

   - D, U *are almost surely increasing.*
   - dD, dU *are almost surely equal to zero on the interval* $(\underline{r}, \overline{r})$*.*
   - *The equilibrium public discount rate* r^{eq} *is given by:*

$$
dr_t^{eq} = \mu_r(r_t^{eq})dt + \sigma_r(r_t^{eq})dW_t + dD_t - dU_t
$$

   and satisfies $r_t^{eq} \in [\underline{r} + \mu_L, \overline{r} + \mu_L]$ *for any* t*.*

We intuitively outline the properties that characterize an equilibrium. Competition in the private sector implies that the actual real estate will earn a constant expected return r^*, which means that the equilibrium value of real estate will be the present value of the cash flows, discounted at the rate r^*. Moreover, for high values of the public discount rate, it becomes optimal for the public companies to sell real estate and repurchase shares, and in so doing, REIT prices increase and public discount rate decrease. The response of the public discount rates to REIT conversions is formally described in our model by the stochastic process dU, which describes how much and how fast the public discount rate is pushed down into the non-conversion interval.

The system characterizing the equilibrium in the Theorem, in particular the first of the set of equations (7), cannot be solved in closed form in general[5] and for this reason we describe how to evaluate the solution numerically in the Appendix.

In order to calibrate the model, we use values of the parameters detailed in Table 1. These values ensure that transaction costs, as well as the first two moments of the private as well as public real estate returns, as implied by the model, are taking values that were observed in the data. In particular, our model produces values for the rates at which it is optimal for an asset to switch its type. These conversion rates, along with the conversion costs associated with them are presented in Table 2. The simulated moments, as implied by our model, are reported in Table 3.

3.4 Comparative Static Analysis of Equilibrium Conversion Policies

Using the model calibrated as described above, we can numerically analyze the sensitivity of the equilibrium conversion bounds $\{\underline{r}, \overline{r}\}$ with respect to model parameters such as the volatility of the public discount rate, the volatility of the cash flows, and the growth rate of the cashflows.

Figure 3 shows that when rate $\overline{r}$ at which REITs sell assets is increasing as a function of the volatility of the public discount rate, while the rate $\underline{r}$ at which REITs purchase private assets is decreasing with the public rate volatility. Therefore, as the public rates are more volatile, the length of the "no-conversion" interval increases. Standard real option intuition accounts for this behavior. For example, given that fixed transaction costs are incurred when real estate is purchased by REITs, REITs may be viewed as holding a call option on the underlying discounted profits from

[5] This variational system admits a closed form solution for lognormal and Ornstein-Uhlenbeck rates of returns and constant flow. It does not have a closed form solution for the case modeled here, that of a CIR process. We present a numerical solution which is applicable to all of these cases in the appendix. For an example of a model with similar analytical characteristics that is solved in closed form we refer the reader to Cherkes, Sagi, and Stanton (2010).

purchasing a real estate asset. The timing of exercise depends on the relative benefit of waiting to exercise the option, in the hope that discount rates will fall further, and purchasing immediately and receiving rents sooner. Optimal exercise in our setting is the purchase of real estate from private soruces, and at the endogenous level $\underline{r}$ these forces balance. Increasing the parameter σ_r increases the value of waiting, and hence we see a reduction in this switching point. The sale of real estate from REITs to private investors can similarly be viewed as a put option and increasing σ_r results in a higher value of waiting to sell, increasing the upper bound on the public discount rate $\overline{r}$.

Figure 4 show how the conversion rates change as a function of the cash flows growth rate. As growth rates increase, the “no-conversion” interval becomes wider. This again follows from considering the relative benefits and costs of waiting to exercise the option to purchase or sell. Increases in the parameter μ_L cause the inaction region for REITs to expand because waiting trades off low current cash flows with potentially higher future cash flows.

3.5 The Relationship between Prices of Publicly- and Privately-held Real Estate

We can now proceed to analyze the relationship between real estate prices and REIT prices.

3.5.1 Relative value of REITs vs Real Estate

Proposition 1 *If REITs can both buy and sell properties, then there exists public discount rates* r_t^{eq} *for which public companies trade at a premium relative to their private counterparts and rates at which public companies trade at a discount. Also, for each* t *there is at least one value* $r_p \in (\underline{r}, \overline{r})$ *such that if* $r_t = r_p$ *REITs trade at par.*

3.5.2 REITs and Private Real Estate Return Integration

The next proposition characterizes the equilibrium returns of public and private real estate companies.

If $r_t \in (\underline{r}, \overline{r})$, the returns of dR^R and dR^P of REITs and property satisfy:

$$
\begin{aligned}
dR_t^R &= r_t^{eq} dt + \left(\sigma_L dW_t^L + \sigma_r (r_t^{eq} - \mu_L) \frac{\hat{\phi}_1'(r_t^{eq})}{\hat{\phi}_1(r_t^{eq})} dW_t^r\right) \\
dR_t^P &= r^* dt + \sigma_L dW_t^L.
\end{aligned}
\tag{10}
$$

Taking expectations:

$$
\begin{aligned}
\mathbb{E}\left[dR_t^R\right] &= r_t^{eq} dt \\
\mathbb{E}\left[dR_t^P\right] &= r^* dt.
\end{aligned}
\tag{11}
$$

In particular, from (11),

$$
\mathbb{E}\left[dR_t^P\right] = r^* dt + \mathbb{E}\left[dR_t^P\right] - r_t^{eq} dt. \tag{12}
$$

Equation (12) implies a particular link between the returns of REITs and property: specifically, equation (12) suggests that the returns of real estate $\mathbb{E}\left[dR_t^P\right]$ can be explained by the returns $\mathbb{E}\left[dR_t^R\right]$ of REITs, however not completely. In order to fully explain the returns of private companies we would need to add the public discount rates $r_t^{eq} dt$ as an explanatory variable in a regression of private returns on public returns. This result suggests that when regressing private returns $PREret$ on public returns $REITret$ and the public discount rate $PUBrate$:

$$
PREret_t = a + bREITret_t + cPUBrate_t + \epsilon_t, \tag{13}
$$

we should have:

- a constant term with *positive* sign, namely, $a =^* dt$ above.
- the coefficient of public b returns will be *positive*.
- the coefficient of public discount rates r^{eq}, c, will be *negative*.

3.5.3 Return Correlations

The instantaneous conditional correlation between REIT and Private Real Estate returns is given by:

Proposition 2 *Denote by*

$$a(r) \quad := \sigma_r(r)\frac{\hat{\phi}_1'(r)}{\hat{\phi}_1(r)}. \tag{14}$$

With these notations:

$$\begin{aligned} dR_t^R &= r_t^{eq}dt + \left(\sigma_L dW_t^L + a(r_t^{eq}dW_t^r\right) \\ dR_t^P &= r^* dt + \sigma_L dW_t^L. \end{aligned} \tag{15}$$

The conditional correlation $\sum(r) = corr\left(dR_t^P, dR_t^R \mid r_t^{eq} = r\right)/dt$ *is then given by:*

$$\sum(r) = \frac{\sigma_L + \rho a(r)}{\sqrt{\sigma_L^2 + a(r)^2 + 2\rho\sigma_L a(r)}}. \tag{16}$$

Figure 5 presents the correlations when the public discount rate takes different values. At times when conversions occur, REITs and private real estate are perfectly correlated, and consequently there is no additional diversification benefits from holding both types of assets simultaneously. However, the correlation decreases to around 0.75 in the middle of the no-conversion interval.

4 Marking-to-Market: Issues and Solutions

REITs and real estate share the common cash flow component, L_t, but differ in price due to differences in required rates of return. As shown in the equilibrium of Section 3, the difference between private and public discount rates is a stationary process. This implies that REIT prices will contain pertinent information for real estate valuation, and in particular for marking to market.

We begin by showing in general how REIT prices relate to private values. Let $v_t^i = \ln\left(V_t^i\right)$ for $i \in \{P, R\}$. Then

$$\begin{aligned} v_t^P &= l_t - \Phi \\ &= l_t - \Phi_t + (\Phi_t - \Phi) \\ &= v_t^R + \eta_t, \end{aligned} \tag{17}$$

where $l_t = \ln(L_t)$, $\Phi_t = \ln(\phi_t)$, $\Phi = \ln(r^* - \mu_L)$, and $\eta_t = \Phi_t - \Phi$. The equilibrium produces a well-defined unconditional distibution of η_t. Under parameterizations of the model in which the mean of η_t is small, equation (17) shows that log REIT values will provide estimates of log private values that are approximately unbiased in an unconditional sense. Marking to market can then be achieved by appropriately accounting for the Jensen inequality effect that arises when the non-linear relationship $V_t^P = e^{v_t^P}$ is applied.

More precise estimates of mark to market values can be produced if, in addition to comparable REIT prices, estimates of private real estate holdings from an appraisal process are available. We assume that appraised values are lagged and smoothed versions of the underlying private real estate value. This would occur, for instance, if the appraiser capitalizes forecasted income reported in historical documents at the private discount rate but does not fully incorporate information on changes in cashflows. To capture this intuition, we model appraisal prices as

$$\begin{aligned} \hat{v}_t^P &= (1-\alpha)v_{t-1}^P + (1-\alpha)\left[\alpha v_{t-2}^P + \alpha^2 v_{t-3}^P + ...\right] \\ &= \alpha \hat{v}_{t-1}^P + (1-\alpha)v_{t-1}^P, \end{aligned} \tag{18}$$

where $0 \leq \alpha < 1$ quantifies the amount of old information remaining in the appraisal and where the equally spaced discrete historical appraisal dates are $\{t-1, t-2, ...\}$.

This appraisal information is critical for producing a mark-to-market value that contains more information than REIT prices alone. The historical time series of appraisals $\hat{v}_t^P$ serves two purposes. First, changes in these values allow estimation of the otherwise unobservable parameter α. Second, this data, along with lagged REIT prices, allows identification of otherwise unobservable lagged public discount rates.

To estimate the smoothing parameter α, note that first differencing the series $\{\hat{v}_t^P\}_{t=1}^{\infty}$ produces capital returns $\hat{r}_t^P = \hat{v}_t^P - \hat{v}_{t-1}^P$ that are autocorrelated with one lag

$$\begin{aligned} \hat{r}_t^P &= \alpha \hat{r}_{t-1}^P + (1-\alpha)[\mu_L + \sigma_L \epsilon_t] \\ &= (1-\alpha)\mu_L + \alpha \hat{r}_{t-1}^P + (1-\alpha)\sigma_L \epsilon_t \end{aligned} \tag{19}$$

where ϵ_t is an independent standard normal random variable. This implies that ordinary least squares can be used to provide a consistent and efficient estimate $\hat{\alpha}$. Equation (18) can then be used to determine an estimate of the unsmoothed lagged private value, given by

$$\begin{aligned} v_{t-1}^P &= l_{t-1} + \Phi \\ &= \tfrac{\hat{v}_t^P - \hat{\alpha}\hat{v}_{t-1}^P}{1-\hat{\alpha}}. \end{aligned} \tag{20}$$

The log of lagged REIT prices is given by $v_{t-1}^R = l_{t-1} + \Phi_{t-1}$, where $\Phi_{t-1} = \ln(\phi_{t-1})$. The difference between this value and the estimate of private value from equation (20) can now be used to produce an estimate of the lagged log discount rate

$$v_{t-1}^R - \frac{\hat{v}_t^P - \hat{\alpha}\hat{v}_{t-1}^P}{1-\hat{\alpha}} = \Phi_{t-1} - \Phi. \tag{21}$$

This equation can be used to generate the entire time-series of the difference between public and private discount rates.

The model in Section 3 provides an explicit relationship for the dynamics of $\Phi_{t-1} - \Phi$ with parameters (e.g., k_r, $\tilde{r}$, and σ_r) that can be estimated by using, for example, the simulated method of moments.

To mark-to-market the private holdings, we require a forecast of $\Phi_t - \Phi$, which can be summarized by the conditional expectation $E(\Phi_t - \Phi|\Phi_{t-1} - \Phi)$. The log mark-to-market valuation is then given by

$$\begin{aligned} v_t^P &= v_t^R - (\Phi_t - \Phi) \\ &= v_t^R - E(\Phi_t - \Phi|\Phi_{t-1} - \Phi) - \eta_t, \end{aligned} \tag{22}$$

where η_t is a zero mean independent random variable. The value $v_t^R - E(\Phi_t - \Phi|\Phi_{t-1} - \Phi)$ therefore provides a conditionally unbiased estimate of the private real estate value. The distribution of η_t can further be exploited to provide, for example, confidence intervals on this value.

5 Calibration and empirical results

In this section we explore the relationship predicted by equation (11) using real data. We first describe the data then we proceed to our tests.

5.1 Data

We use data from the Ziman REIT database, from which we create separate portfolios based on different property types. We also examine quarterly commercial real estate returns from NCREIF, where we also separate the companies by property type while building representative indices. The reason to use property types instead of just indices containing all the companies is that the weights of property types in NCREIF differ from those in the Ziman database, thus the characteristics of the companies included in the public, respectively private indices built using all properties will be different. Since NCREIF returns are reported without leverage while the REITs returns include leverage, we first use REIT debt information from COMPUSTAT and de-lever REIT returns, assuming that their debt has a BAA credit rating. Summary statistics are presented in Table 5. From the Panel A of Table 5 we observe that the means and standard deviations of public and private property indices are consistent with our simulations. The standard deviations of the private property indices are somewhat lower than those reported before, for example by Riddiough, Moriarty, and Yeatman (2005). We calibrated our model to be closer to observed standard deviations, rather than the (higher) standard deviations reported in Riddiough, Moriarty, and Yeatman (2005). Panel B presents serial correlation coefficients for the Private Real Estate indices. From Panel B we observe that these returns are positively autocorrelated to up to four lags.

5.2 The Link between Private and Public Real Estate in the Data

In this subsection we test the main prediction of our model in Section 3, namely, that the returns of private real estate are equal to the returns of public real estate minus the instantaneous public discount rate.

As in our simulated results, we first start by regressing the returns of private real estate on public real estate, then adding a proxy of the public discount rate as an explanatory variable. The proxy we use for the public discount rate is the return of the Fama-French SMB index. Furthermore, our model is in real terms, while our indices are nominal. For this reason, we also introduce the yields on 10 year Treasury Bills as a control in the equations. Furthermore, returns on private real estate are smoothed to up to four lags (see Panel B of Table 5). Thus, as independent variables we must not only include contemporaneous values of the returns on public real estate, the public interest rate proxy and yields on treasuries, but also their lagged values. Since our data frequency is quarterly, it is unrealistic to consider all the lags (if we do so we end up with 15 independent variables). Thus, instead of separately employ each lag from lag 1 to lag 4, we use their equally weighted average.

These results are reported in Table 6. We observe the following:

- As in our simulated results for the case in which private returns were smoothed, attempting to explain private returns using public returns has very little success. In Table 6 all the adjusted

R-squareds from these regressions of private returns on contemporaneous REIT returns are relatively small - typically under 15%.

- When the public discount rate (proxied by SMB) is introduce the R-squared improves (except for the case of Retail properties). The coefficients of the public discount rate appear with the negative sign consistent with the prediction of our model's equation (12), and are significant.
- The public discount rate also appears with a negative coefficient when its average lag is considered. Except for Retail and Apartments the coefficient on the lag of the public discount rate SMB is also significant (and negative).
- Whereas the initial R-squareds of the regressions of private on public returns where dismal, after accounting for the public discount rate, for autocorrelations in NCREIF returns and for the fact that our indices prices are nominal, the R-squareds increase to as much as close to 57%.

6 Conclusions

The relation between actual real estate price and REIT prices has attracted the attention of both industry participants and academics for a number of years. Academics have noted that because the REIT market is much more efficient and liquid than the property market, observed REIT prices may provide a more accurate picture of the property market than one might get from looking at potentially stale appraisals. A number of industry participants have noted, however, REIT prices are influenced by stock market factors that may have nothing to do with what is happening in the property markets, and as a result, they may provide unreliable indicators of property market values.

The model developed in this paper provides a framework for understanding the co-movements of real estate and REIT prices. As the model illustrates, because the required rates of return on real estates and REITs often differ, REITs can sell for a discount or a premium relative to the real estate they hold. In addition, we expect to see positive but imperfect correlation between real estate and REITs, and the correlation will vary over time in predictable ways.

Our model has practical applications that will be explored in more detail in our ongoing research. Most of these applications relate to FASB 157 that requires institutional investors to mark to market their less liquid investments. As a result, there is an increased interest in utilizing public market prices to value privately held real estate, and in addition, to better understand how private real estate contributes to the overall risk of the investor's portfolio. As we show, one can obtain a more accurate estimate of real estate values by combining possibly stale appraisal values, REIT returns, and other stock returns than one can obtain by just looking at either appraised values or REIT prices in isolation.

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Appendix 1: Proofs

Proof of the Equilibrium characterization theorem

First, we note that since the supply of private real estate is perfectly elastic, the discount rate for private real estate is always r^* and in turn, the price of private real estate is always:

$$V^P(r, L) = \mathbb{E}\left[\int_0^\infty e^{-r^* t} L_t dt | L_0 = l, r_0 = r\right] = L/(r^* - \mu_L). \tag{23}$$

We can now turn to the REITs price $V^R(r, L)$. By Bellman's optimality principle, for any (small) $\theta > 0$ we have:

$$V^R(r, L) = \sup \mathbb{E}\left[\int_0^\theta e^{-\int_0^t R_s(I_s)ds} L_t dt - \sum_{\tau_n \le \theta} e^{-\int_0^{\tau_n} R_s(I_s)ds} c_{k_{n-1},\ k_n} L_{\tau_n} + e^{-\int_0^\theta R_s(I_s)ds} V^{I_\theta}(r_\theta, L_\theta)\right]. \tag{24}$$

Now, by Ito's lemma,

$$V^{I_\theta}(r_\theta, L_\theta) = V^{I_\theta} + \int_0^\theta \mathcal{L} V^{I_\theta} dt + \int_0^\theta \triangledown V^{I_\theta} \Sigma dW_t, \tag{25}$$

where Σ is the volatility process for (r_t, L_t) jointly, $W = (W^r, W^L)'$ is the Brownian motion driving (r_t, L_t) and the differential operator $\mathcal{L}$ is:

$$\mathcal{L} f(r, L) = (1/2)\sigma_r^2 f_{rr} + \rho \sigma_r \sigma_L f_{rL} + (1/2)\sigma_L^2 f_{LL} + \mu_r r_f + \mu_L f_L.$$

Since the expectation of the Brownian integral is zero, the Bellman's optimality principle becomes:

$$\begin{aligned} \inf \mathbb{E}\big[V^R(r, L) \quad & -e^{-\int_0^\theta R_s ds} V^{I_\theta}(r, L) - e^{-\int_0^\theta R_s ds} \textstyle\int_0^\theta \mathcal{L} V^{I_\theta} dt \\ & - \textstyle\int_0^\theta e^{-\int_0^t R_s ds} L_t dt + \sum_{\tau_n \le \theta} e^{-\int_0^{\tau_n} R_s(I_s)ds} c_{k_{n-1},\ k_n} L_{\tau_n}\big] = 0. \end{aligned} \tag{26}$$

We now distinguish three possibilities. In the first, it is optimal to switch from REITs to private real estate at the current time. If this is the case, then as we push $\theta \to 0$,

$$V^R(r, L) = V^P(r, L) - c_{RP} L. \tag{27}$$

In the second, we just switched from the private state and thus:

$$V^R(r,L) = V^P(r,L) + c_{PR}L. \tag{28}$$

The third possibility is that it is optimal to have had no switches at the current time. In this case, for a small enough θ, $I_\theta = R$ and there are no switching costs being paid between 0 and θ, i.e.,

$$\sum_{\tau_n \le \theta} e^{-\int_0^{\tau_n} R_s(I_s)ds} c_{k_{n-1},\ k_n} L_{\tau_n} = 0.$$

Dividing by θ in (26), and pushing θ to zero, we have:

$$\lim_{\theta\to 0} \frac{V^R - \exp(-\int_0^\theta R_s ds)V^{I_\theta}}{\theta} = \lim_{\theta\to 0}\left[\frac{1-\exp(-\int_0^\theta R_s ds)}{\theta}\right] V^R,$$

where the equal sign follows because it is suboptimal to change type before time θ. The above limit is further equal to

$$V^R \lim_{\theta\to 0} R_\theta(r)\exp(-\int_0^\theta R_s ds) = rV^R(r,L).$$

We also have that

$$\lim_{\theta\to 0} -e^{-\int_0^\theta R_s ds}\frac{\int_0^\theta \mathcal{L}V^{I_\theta}dt}{\theta} = -\mathcal{L}V^R(r,L)$$

and that

$$\lim_{\theta\to 0}\frac{-\int_0^\theta e^{-\int_0^t R_s ds}L_t dt}{\theta} = -l.$$

Therefore, if switching is suboptimal,

$$rV^R - \mathcal{L}V^R - L = 0. \tag{29}$$

This equation can be rewritten as:

$$\begin{aligned}&-(1/2)\sigma_r^2 V_{rr}^R - \rho\sigma_r\sigma_L L V_{rL}^R - (1/2)\sigma_L^2 L^2 V_{LL}^R \\ &-\mu_r V_r^R - \mu_L L V_L^R + r^* V^R - L = 0.\end{aligned} \tag{30}$$

We can look for a solution of the form $V^R(r,L) = L\phi_R(r)$, and substituting in (30) we observe that ϕ^R satisfies:

$$\sigma_r\phi_R'' - (\rho\sigma_r\sigma_L + \mu_r)\phi_R' - (\mu_l - r)\phi_R - 1 = 0. \tag{31}$$

At the the point $\overline{r}$ such that at $(\overline{r}, L)$ it is optimal to switch from public to private for some L, we have that

$$\phi_R(\overline{r}) = 1/(r^* - \mu_L) - c_{RP}. \tag{32}$$

Furthermore, at $\underline{r}$ such that it is optimal to switch from private to public at $(\underline{r}, L)$, we have that

$$\phi_R(\underline{r}) = 1/(r^* - \mu_L) + c_{RP}. \tag{33}$$

Finally as in Dumas (1991) at the switching points we must have smooth pasting conditions. Given that the price of the private assets is proportional to L, at each of the optimal switching points we have that the first order derivatives of the function ϕ_R are null, i.e.,

$$\phi_R'(\underline{r}) = 0 \;\text{ and }\; \phi_R'(\overline{r}) = 0. \tag{34}$$

While exact solutions for the CIR Kolmogorov equation exist, equation (31) is different. In order to find a solution, we can proceed numerically.

In order to calculate a numerical solution, we proceed in the following steps:

1. For an arbitrary number $r_2 > 0$, we solve equation (7) numerically the resulting ODE on the interval $[0, r_2]$, with the boundary conditions $\phi^R(r_2) = 1/(r^* - \mu_L) - c_{RP}$ and $(\phi^R)'(r_2) = 0$. The solution will be decreasing.

2. Find out the point r_1 (which depends on r_2) such that

$$r_1 = \arg\max\{(\phi_R(r) - 1/(r^* - \mu_L) - c_{PR})^2\}.$$

3. Find that r_2 such that $\phi_R(r_1) = 1/(r^* - \mu_L) + c_{PR}$. Take $\overline{r} := r_2$. Find the r_1 associated with $r_2 = \overline{r}$. Take $\underline{r} := r_1$.

4. If $r_1 = 0$ and ϕ_R has a strictly negative derivative at $r_1 = 0$, then no conversions from private to public occur. If $r_1 > 0$ (i.e. r_1 is an interior maximum) then conversions from private to public occur at r_1.

Proof of Proposition 1:

Since boundary conditions are satisfied at $\underline{r}$ and $\overline{r}$, we observe that

$$\phi_R(\underline{r}) > 1/(r^* - \mu_L) > \phi_R(\overline{r}).$$

Therefore, since ϕ_R is continuous, there is a point r^p such that $\phi_R(r^p) = 1/(r^* - \mu_L)$.

Derivation of equation (10):

The proposition follows from the fact that:

$dR_t^R = \frac{d[L_t\phi_R(r_t^{eq})]+L_t dt}{L_t\phi_R(r_t^{eq})}$, $dR_t^P = \frac{dL_t+L_t dt}{L_t}$ and from the Itô's lemma.

Tables and Figures

Table 1: Calibration parameters

The Table presents the values used to calibrate the model of Section 3.

Variable	Notation	Value used in calibration
Private discount rate	r^*	10.00%
Public discount rate model	r_t	$dr_t = k_r(\tilde{r} - r_t)dt + \sigma_r \sqrt{r_t} dW_t^r$
	k_r	0.01
	$\tilde{r}$	2.00%
	σ_r	0.13%
Flows	L_t	$dL_t = \mu_L L_t dt + \sigma_L L_t dW_t^L$
		$dW_t^r \times dW_t^L = \rho dt$
	μ_L	6.00%
	σ_L	10.00%
	ρ	0.80
Costs of switching $C = cL$		
From Public to Private	c_{12}	1.6
From Private to Public	C_{21}	1.5

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Table 2: Calibration parameters

The Table presents the results obtained after solving the model of Section 3.

Conversion costs (in %)	
From Public to Private	6.04%
From Private to Public	6.41%
Optimal Conversion Rates	
From Private to Public $\underline{r}$	5.46%
From Public to Private $\overline{r}$	16.36%

Table 3: Summary statistics from simulation.

The Table present summary statistics of public and private real estate obtained from simulating prices as given by Section 3. We ran 10,000 simulations. In each simulation, we considered 1,000 quarters for the public discount rate r starting with $r_0 = (\underline{r} + \overline{r})/2$. For each simulation we calculate the returns $d\tilde{R}_{it}$ of public and private companies and their correlation as in equation (10) and recorded the time series means and standard deviations of these returns. Private real estate returns were smoothed using an MA(3) specification. The Table presents averages of the 10,000 estimations of time series averages and standard deviations (annualized) for public and private real estate returns. The standard deviations of the 10,000 estimated time series means and standard deviations are reported in parentheses.

Public Real Estate		Private Real Estate	
Mean	Stdev.	Mean	Stdev.
5.57%	9.88%	5.47%	4.93%
(0.63%)	(0.22%)	(0.63%)	(0.18%)
Corr.	0.50	(0.01)	

Table 4: Public and Private real estate returns: Simulated Linkage

The Table presents estimations of the model:

$$PRE_t = const. + \beta_{REIT}\ REIT_t + \beta_r\ R_t + \epsilon_t$$

on simulated data. $REIT, PRE$ are the returns of public, respectively private real estate companies and the public R is the public instantaneous discount rate. The equation tested is inspired by equation (12) The returns to private real estate may be smoothed, and if this is the case then the observed returns are period t is the equally weighted average of the actual returns at periods $t, t-1, ..., t-Nlags$.

Nobs.	const. (tstat)	β_{REIT} (tstat)	β_r (tstat)	$\overline{R}^2$	const. (tstat)	β_{REIT} (tstat)	β_r (tstat)	$\overline{R}^2$	const. (tstat)	β_{REIT} (tstat)	β_r (tstat)	$\overline{R}^2$	const. (tstat)	β_{REIT} (tstat)	β_r (tstat)	$\overline{R}^2$
	$Nlags = 0$				$Nlags = 1$				$Nlags = 3$				$Nlags = 4$			
80	-0.00	0.79			0.01	0.41			0.01	0.12			0.02	0.12		
	(-0.39)	(21.94)		86.03%	(1.69)	(6.81)		37.10%	(4.51)	(2.51)		6.63%	(5.22)	(2.95)		9.40%
	0.05	0.73	-1.11		0.08	0.26	-1.48		0.06	0.12	-1.04		0.04	0.09	-0.64	
	(19.42)	(51.68)	(-21.09)	97.93%	(8.83)	(5.72)	(-8.52)	67.59%	(11.62)	(3.62)	(-9.71)	58.72%	(6.27)	(2.49)	(-4.42)	27.77%
120	0.00	0.78			0.01	0.28			0.02	0.13			0.02	0.10		
	(0.97)	(32.32)		89.84%	(4.84)	(6.67)		27.10%	(7.83)	(3.99)		11.50%	(7.54)	(3.72)		10.11%
	0.05	0.74	-1.06		0.06	0.22	-1.09		0.04	0.11	-0.56		0.04	0.09	-0.63	
	(23.40)	(74.34)	(-24.25)	98.31%	(8.59)	(6.32)	(-7.04)	48.60%	(6.11)	(3.46)	(-3.34)	38.73%	(8.57)	(3.64)	(-5.84)	30.45%
240	0.00	0.81			0.01	0.31			0.01	0.13			0.01	0.13		
	(3.90)	(48.32)		90.75%	(5.89)	(8.85)		24.61%	(7.17)	(4.56)		7.76%	(7.80)	(5.32)		10.46%
	0.05	0.74	-1.04		0.07	0.21	-1.27		0.06	0.06	-0.94		0.05	0.09	-0.75	
	(33.04)	(95.93)	(-31.88)	98.25%	(14.89)	(7.75)	(-12.90)	55.68%	(16.00)	(2.83)	(-13.38)	47.62%	(12.85)	(4.28)	(-10.03)	37.26%
300	0.01	0.81			0.00	0.31			0.02	0.16			0.02	0.12		
	(6.46)	(48.70)		88.83%	(2.16)	(10.71)		27.69%	(9.43)	(6.46)		12.12%	(10.47)	(5.09)		7.80%
	0.05	0.75	-1.07		0.06	0.26	-1.12		0.07	0.10	-1.16		0.06	0.06	-0.90	
	(38.68)	(101.03)	(-35.71)	97.89%	(13.75)	(11.25)	(-13.62)	55.44%	(19.44)	(5.49)	(-15.80)	52.38%	(16.09)	(3.18)	(-12.15)	38.54%

Table 5: Summary statistics for public and private real estate.

The Table present summary statistics of public and private real estate. The public real estate are Real Estate Investment Trusts from the Ziman database at the University of California at Berkeley. The returns of each REIT are de-levered using data on their debt to equity ratios from COMPUSTAT and assuming that their debt has a BAA rating. The private real estate indices represent total returns on properties from NCREIF. Panel A reports means and standard deviations of the public and private indices while Panel B reports autocorrelation coefficients in the returns of Private Real Estate indices, estimate by regressing contemporaneous returns of Private Real Estate indices on a constant term and their lags. The t-statistics of these coefficients are in parentheses. The property types are from NCREIF. *Office* and *Industrial* corresponds to property type *Industrial/Office* from Ziman. *Retail* corresponds to the *Retail* type in Ziman. *Hotel* corresponds to the *Lodging/Resorts* type in Ziman. *Apartments* corresponds to the *Residential* type in Ziman. All REITs are equity REITs. The data spans the horizon 1988 Q2 to 2009 Q1.

Panel A : Means and Standard Deviations of Public and Private Real Estate

	Public (Ziman unlevered)		Private (NCREIF)	
	Mean	Stdev.	Mean	Stdev.
Office	6.69 %	11.07 %	6.36 %	5.88 %
Industrial	6.69 %	11.07 %	7.92 %	4.66 %
Retail	7.85 %	11.01 %	8.38 %	4.22 %
Hotel	2.37 %	15.79 %	8.70 %	6.71 %
Apartments	8.06 %	8.73 %	8.65 %	4.24 %

Panel B : Autocorrelation in Private Real Estate returns

	const.	Lag1	Lag2	Lag3	Lag4	Lag5
Office	0.0011	0.6757	0.3216	0.0310	0.4428	-0.6186
	(0.4231)	(6.5410)	(2.4734)	(0.2274)	(3.3863)	(-4.9358)
Industrial	0.0012	0.7471	0.3611	0.0684	0.3613	-0.6532
	(0.4769)	(7.1321)	(2.3789)	(0.4265)	(2.3501)	(-4.5887)
Retail	0.0016	0.6026	0.3845	-0.0617	0.2887	-0.3406
	(0.5878)	(5.3903)	(2.6749)	(-0.4376)	(2.1429)	(-2.6850)
Hotel	0.0043	0.4739	0.2095	0.1406	-0.0708	0.0104
	(0.8596)	(3.9875)	(1.4615)	(0.9691)	(-0.4934)	(0.0761)
Apartments	0.0005	0.8442	0.3798	0.0377	0.1454	-0.4736
	(0.1337)	(7.5438)	(2.0763)	(0.1935)	(0.7960)	(-2.8524)

Table 6: The link between public and private returns in the data

The Table presents estimations of the model:

$$PRE_t = const. + \beta^0_{REIT}\ REIT_t + \beta^0_r\ R_t + \beta^0_T reasTreas10_t + \beta^1_{REIT}\ REIT_{t-4:t-1} + \beta^1_r\ R_{t-4:t-1} + \beta^1_T reasTreas10_{t-4:t-1} + \epsilon_t$$

on public and private real estate indices from the Ziman and respectively NCREIF databases. $REIT_t, PRE_t$ are the returns of public, respectively private real estate companies in quarter t, R_t is the proxy for the instantaneous discount rate (we use the Fama-French SMB portfolio returns of quarter t). The returns with subscripts $t-4:t-1$ represent the average returns of the respective index in quarters $t-4, t-3, t-2$ and $t-1$. The model is that of equation (12). The t-statistics are Newey-West corrected for serial correlation in variables. Data spans the time period 1988 Q2 to 2009 Q1.

Table 6 (cont.): The link between public and private returns

		$REIT_t$	SMB_t	$Treas10_t$	$REIT_{t-4:t-1}$	$SMB_{t-4:t-1}$	$Treas10_{t-4:t-1}$	$\bar{R}^2$
Office	Coef.	0.1889						
	t-stat	2.1514						11.34%
	Coef.	0.2228	-0.1166					
	t-stat	2.5853	-2.3705					14.51%
	Coef.	0.2339	-0.1944	-0.3071				
	t-stat	4.3960	-3.8531	-3.2413				28.24%
	Coef.	0.1559	-0.0809		0.5008	-0.2752		
	t-stat	2.6575	-1.9985		3.5561	-2.1577		33.87%
	Coef.	0.1378	-0.1063	-0.1971	0.4346	-0.3421	-0.6527	
	t-stat	2.9759	-2.6067	-3.6532	4.6509	-2.8789	-3.0502	52.33%
Ind	Coef.	0.1764						
	t-stat	2.1679						16.27%
	Coef.	0.2027	-0.0904					
	t-stat	2.4905	-2.5034					19.32%
	Coef.	0.2119	-0.1548	-0.2542				
	t-stat	3.9448	-3.6307	-2.9302				34.48%
	Coef.	0.1483	-0.0620		0.3973	-0.1849		
	t-stat	2.7191	-2.4169		3.3434	-1.9174		37.95%
	Coef.	0.1354	-0.0860	-0.1696	0.3384	-0.2352	-0.5158	
	t-stat	3.2062	-2.9218	-3.3221	4.9887	-2.4981	-3.3641	57.53%
Retail	Coef.	0.0978						
	t-stat	1.2212						5.66%
	Coef.	0.1025	-0.0163					
	t-stat	1.2124	-0.5362					4.61%
	Coef.	0.1417	-0.0856	-0.2380				
	t-stat	2.3129	-2.0767	-2.7717				20.71%
	Coef.	0.0717	-0.0145		0.3208	-0.0185		
	t-stat	1.2031	-0.4933		2.4721	-0.2103		17.77%
	Coef.	0.0742	-0.0389	-0.1558	0.3672	-0.1049	-0.5189	
	t-stat	1.7215	-1.1513	-3.4409	5.5157	-1.0792	-2.9405	42.24%
Apt	Coef.	0.1914						
	t-stat	1.5404						14.45%
	Coef.	0.2056	-0.0522					
	t-stat	1.6131	-1.7450					15.09%
	Coef.	0.2274	-0.1242	-0.2809				
	t-stat	2.5752	-2.8021	-2.6963				37.90%
	Coef.	0.1853	-0.0483		0.3746	-0.0391		
	t-stat	1.7891	-1.9939		1.9784	-0.4820		24.67%
	Coef.	0.1897	-0.0963	-0.2391	0.2935	-0.0697	-0.3733	
	t-stat	2.6452	-2.9085	-3.2407	2.6360	-0.8268	-2.7751	49.46%
Hotel	Coef.	0.1337						
	t-stat	1.9647						8.52%
	Coef.	0.1878	-0.1826					
	t-stat	2.8387	-2.6792					14.74%
	Coef.	0.1790	-0.2301	-0.2177				
	t-stat	3.1800	-3.1324	-1.6069				19.40%
	Coef.	0.1211	-0.1224		0.3699	-0.4132		
	t-stat	2.5232	-2.5008		3.6281	-2.9968		37.27%
	Coef.	0.1060	-0.1361	-0.1383	0.3487	-0.4472	-0.4058	
	t-stat	2.1693	-2.7029	-1.4367	4.2773	-3.3746	-1.9766	42.10%

Figure 1: REIT supply and demand.

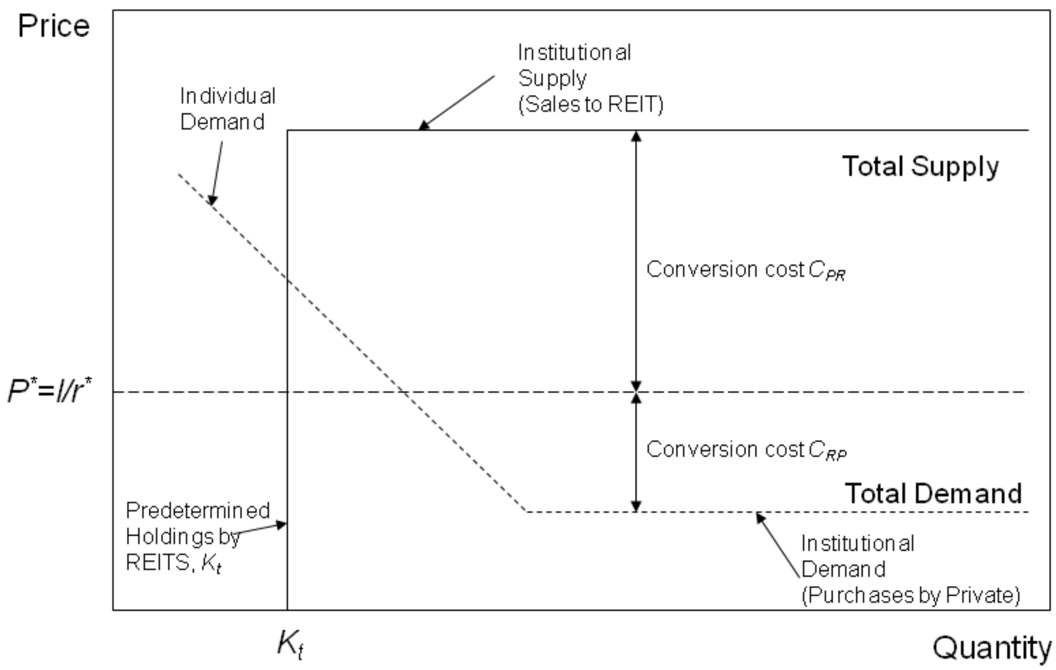

Figure 2: Prices.

The figure presents share prices of public and of private real estate.

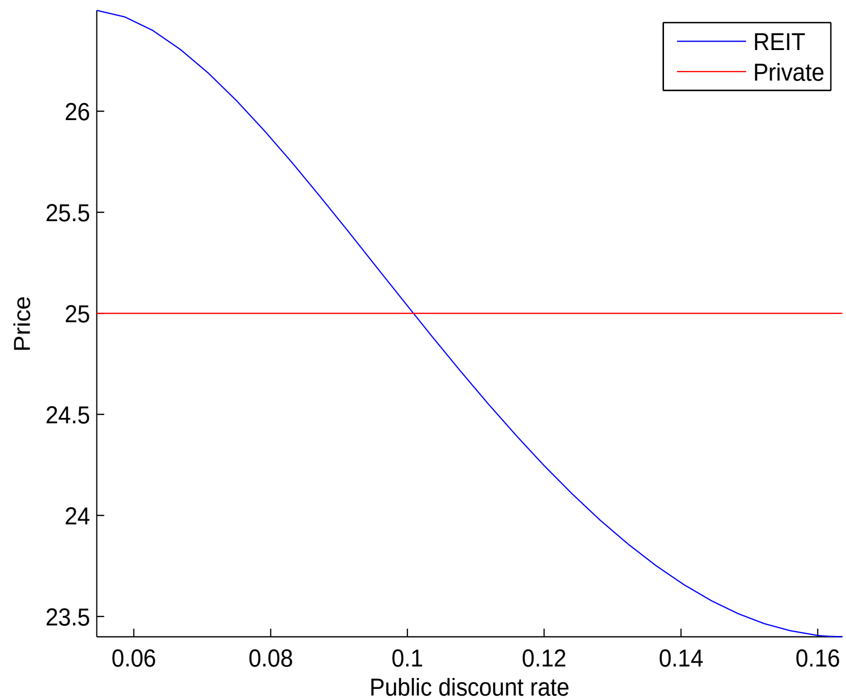

Figure 3: Public rate volatility and switching rates.

The Figure present the switching rates $\overline{r}$ and $\underline{r}$ as a function of the volatility σ_r of the public discount rate.

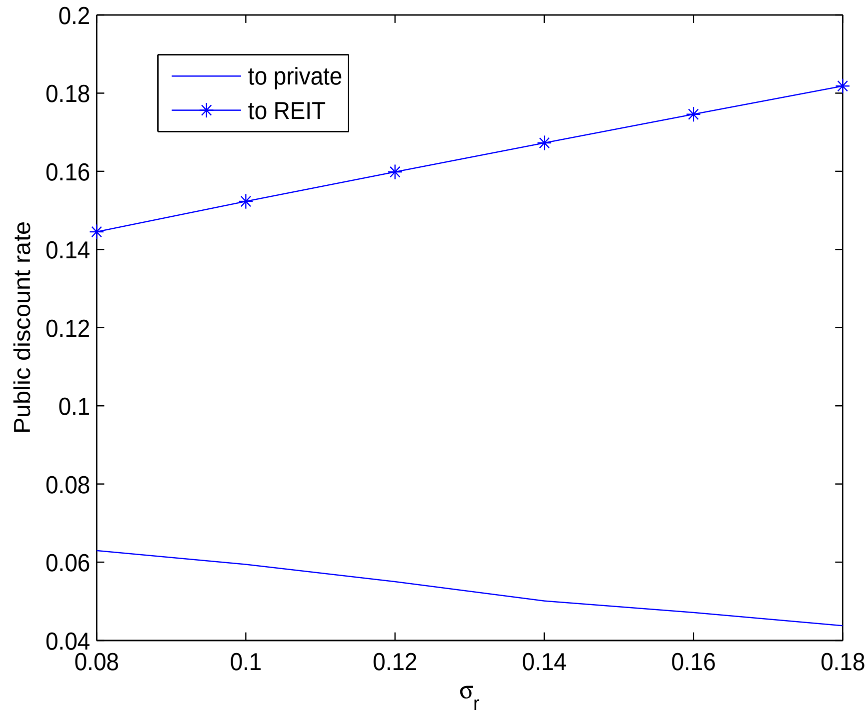

Figure 4: Cash flow growth and switching rates.

The Figure present the switching rates $\overline{r}$ and $\underline{r}$ as a function of the cash flow growth rate μ_L.

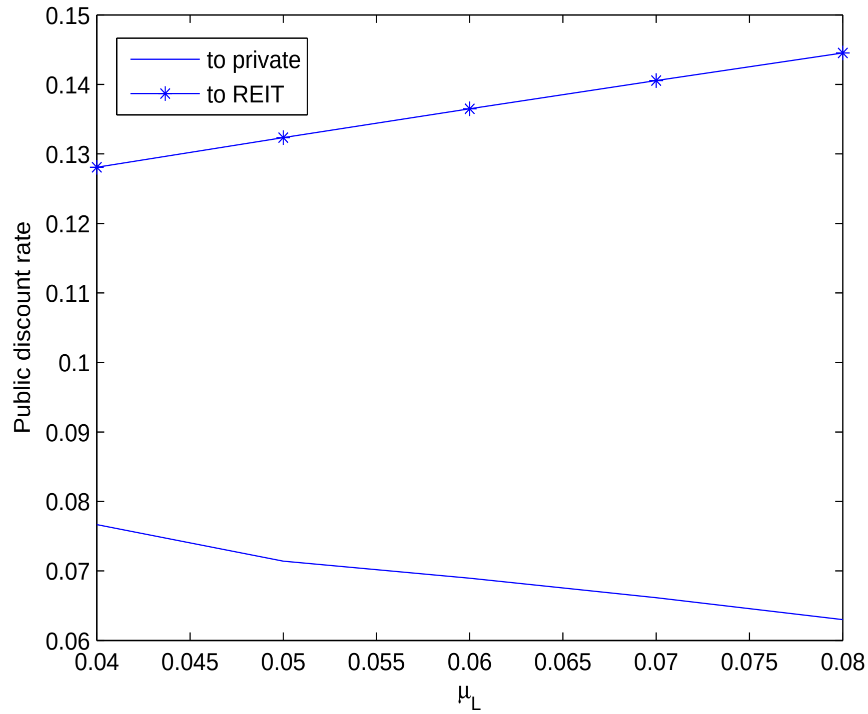

Figure 5: Correlations.

The figure presents the correlation between the returns of public and private real estate for different values of the public discount rate.