Valuing Retail Shopping Center Lease Contracts

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Abstract

In this paper we set up a simple real option model of retail shopping center leases. The model incorporates the effects of stochastic sales externalities and the possibility of tenant default. The model sheds new light on a number of puzzles: Why a Jorgensonian user cost of capital may overestimate shopping center values. Why retail tenants pay their landlord a percentage of revenue (above a sales breakpoint) in addition to a traditional fixed rent. Why an increase in sales volatility may reduce, rather than increase, the value of a shopping center. Why base and percentage rents may move positively together, and not negatively as others have suggested. Why retail property capitalization rates and interest rates can be negatively, as opposed to positively, related with each other. And: Why leases with a percentage of sales rental are often given little or no value for purposes of financing by lenders, regardless of whether or not the tenant is a national chain with substantial net worth.

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1. Introduction

The pricing of retail leases has especially received attention in studies by Benjamin, Boyle and Sirmans (1990, 1992), Brueckner (1993), Colwell and Nunneke (1998), Pashigian and Gould (1998), and Gould, Pashigian, and Prendergast (2005), and by Grenadier (1995, 1996), Hendershott and Ward (2000, 2002), Moordian and Yang (2000), Clapham (2003), and Benjamin and Chinloy (2004). The studies by Benjamin, Boyle and Sirmans (1990, 1992), Brueckner (1993), Colwell and Nunneke (1998), Pashigian and Gould (1998), and Gould, Pashigian, and Prendergast (2005) discuss the pricing of retail leases in a deterministic framework in which consumers are drawn to shop at the center by the reputation and advertising of the major retailer. These models have problems, however, in valuing percentage of sales rentals, rent bumps, expense stops, etc.

The studies by Grenadier (1995, 1996), Hendershott and Ward (2000, 2002), Mooradian and Yang (2000), Clapham (2003), and Benjamin and Chinloy (2004), on the other hand, discuss the pricing of retail leases in a framework in which sales follow a stochastic process. In these models, feedback effects, such as the loss of a major tenant (or a decline in the tenant's customer drawing power) on sales, the center's occupancy rate, and rent levels of the other tenants, are generally ignored. Therefore, these models systematically underestimate risk and overestimate value when default is an important element in the analysis.

The present study integrates these two lines of research. The model developed here treats the stochastic component of store sales in a manner similar to Hendershott and Ward (2000, 2002), and shared and generative business within the shopping center in a manner similar to Brueckner (1993). Additionally, the model allows for any tenant of any size to default, and for default of a key anchor tenant under the lease agreement to lead to further defaults by

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other tenants.

We assume tenants have a claim on all future profits (modeled as a stochastic process) and the market value of the center represents the expectation of these cash flows discounted at a constant risk-adjusted discount rate. Tenants are price-takers in the leasing market, competing with many other tenants in a perfectly competitive market. Rental prices are fixed for the term of the lease at lease commencement. These rental prices vary by type of tenant so as to take into account customer-drawing ability. The single optimization problem is for the tenant to determine when to exercise his or her default option.

The solution to the model follows by applying stochastic dynamic programming. The analysis is performed on a lease by lease basis. The results are summed across all leases to obtain the market value of the center. Different sets of simulation are then carried out to examine the comparative statics of the model and study the impact of shifts in rental rates and the level of return on valuation.

Several new results emerge. One, the usual certainty theory for measuring the user cost of a retail lease as the sum of the expected risk-adjusted discount rate, plus the depreciation incurred and minus the rate of asset price change no longer applies. The user cost also needs to include a risk premium to compensate the owner/tenant for the risk that the sales externality effects at the center could dissipate or disappear over time. Two, and somewhat related, ignoring this risk premium may significantly overestimate shopping center value. Three, the greater the expected volatility in sales, the less valuable is the lease contract to the owner. Others have suggested that the more uncertain the sales, the greater the value of the shopping center. Four, base and percentage rents in our model may move positively together, and not negatively as others have suggested. Five, retail property capitalization rates and interest

rates can be negatively, as opposed to positively, related with each other (as we shall show below). Six, the model reveals a perhaps somewhat latent role for percentage of sales rentals: namely, to offset the negative value to center owners of the tenant's option to cease to operate at the center or, at worst, to default.[1]

But this is not all. The model also helps to explain why leases with a percentage of sales rental are often given little or no value for purposes of financing by lenders. If the role of percentage of sales rentals is, in fact, to offset the negative value to center owners of the tenant's option to default, this would mean that percentage rents and tenant defaults would not have to be taken into account in the valuation, as long as the default-free lease cash flows were discounted at the risk-adjusted rate of return.

The remainder of the paper is organized as follows. The paper first develops a model for the pricing of retail leases that depends on thirteen parameters, one of those being the extent to which individual tenants take advantage of one another as to confer benefits on one another. The paper then describes the initial parameterization of the model. Next, we present a discussion of the simulation results, followed then by conclusions.

2. Model Setup

There are n different tenants at the shopping center. Each tenant signs a (long-term) lease with a fixed base rent, and a percentage rent clause. Tenants let space at the center at time $t = 0$ and remain there until default; there are no tenant default costs. This assumes all leases

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[1] Compare this to Benjamin, Boyle, and Sirmans (1990), and Brueckner (1993): who argue that percentage rents serve as a mechanism to shift risk from susceptible tenants to landlords who can better handle risk; however, for this to happen the optimal contract needs to be in the form of a pure percentage rent and cannot have a fixed rent component (see Lee (1995)). Miceli and Sirmans(1995) offer a substantially different account, in which percentage lease agreements are used to coordinate the behaviors of multiple interdependent tenants through a common agent, the shopping center owner. Wheaton (2000), and Riddiough and Williams (2005) argue that percentage lease agreements are used to align the interest of the shopping center owner with that of the tenants.

have a co-tenancy or kick-out clause, which allows tenants at the center to default without further cost if one of the anchor tenants at the center defaults or ceases to operate, or if sales should fall below a certain level.[2]

Formally, we let $S_i(t)$ stand for sales per square foot for tenant i. Sales are produced with the help of two factors, the amount of space that is occupied by tenant i and the amount of space allocated to the other tenants, according to the constant-elasticity production function $S_i(t) = D_i(t) \cdot X_i(t) Q_i^{\lambda_i} \prod_{j \neq i} Q_j^{\gamma_{ji}}$. We use Q_i and Q_j to denote the amount of space occupied by tenants i and j, respectively. The elasticities of production are λ_i and γ_{ji}, where $\lambda_i < 0$ and γ_{ji} is either negative or positive. For example, a higher $Q_i(t)$ means a lower $S_i(t)$, and a higher $Q_j(t)$ means a higher $S_i(t)$, on condition, obviously, that tenant j confers a positive externality on tenant i. Should $\gamma_{ji} = 0$, tenant j would confer no externality on tenant i and, should γ_{ji} be negative, tenant j would generate a negative externality. We use the following notation, X_i, to denote demand shocks (i.e., something that causes a shift in the production function). We assume $X_i(t)$ is an event that evolves as a geometric Brownian motion, hence $dX_i(t) = \mu_i X_i(t)dt + \sigma_i X_i(t)dZ_i(t)$, where μ_i is the instantaneous conditional expected percentage change in X_i per unit time, σ_i is the instantaneous conditional standard deviation per unit time, and $dZ_i(t)$ represents the increments of a standard Weiner process. We use the notation, $D_i(t)$, to account for normal depreciation. We assume that $D_i(t)$ follows the deterministic function $dD_i(t) = -\theta_i D_i(t)dt$ and $D_i(0) = 1$, where θ_i is the economic rate of depreciation applied to $D_i(t)$.

In certain parts of the paper (for comparison purposes) we will focus on three different

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[2]The one sure way for sales at the center to fall below a certain level is for one of the anchor tenants to default (see Gatzlaff, Sirmans, and Diskin (1994)). However, there are many other variables to consider and this is not the only way that center sales can fall. For example, center sales can fall, and fall hard, if the center finds itself being anchored by the number 3, 4, or 5 grocer in the market and losing market share to Wal-Mart, or if the center finds itself being anchored by an all-around department store and losing market share to "category killers" like Home Depot, Toy-R-Us, or Circuit City.

lease contracts: a pure fixed rent contract, a profit share rent contract (where the tenant pays nothing but percentage rent), and a percentage rent contract (where the tenant pays a base rent plus an overage rent, giving the center owner higher annual rents in years when tenant sales are particularly strong). For that purpose, we provide here the definitions of these

(i) a pure fixed rent contract is given by $P_i^a(t) = \alpha_i$ for all sales, where α equals a fixed base rent.

(ii) a profit share rent contract is given by $P_i^b(t) = \alpha_i + \beta_i S_i(t)$, where β_i is an overage rate, and $\beta_i S_i(t)$ can be viewed as an incentive payment to the center owner.

(iii) a percentage rent contract is given by $P_i^c(t) = \alpha_i + \beta_i[S_i(t) - \bar{S}_i] \cdot I[S_i(t) > \bar{S}_i]$, where $\bar{S}_i$ is a sales threshold level and $I[S_i(t) > \bar{S}_i]$ represents an indicator function, 1 if $S_i(t) > \bar{S}_i$ and 0 otherwise.

We assume (for computational simplicity) that all other operating costs (e.g., property charges such taxed, insurance, and maintenance) are zero.[3]

The approach in the paper is to perform a lease by lease analysis to estimate the dollar value of each lease. We then sum across all leases to obtain the market value of the center.

We assume that the center owner is the developer and landlord. We assume that the center owner has already decided the amount of investment and the optimal tenant mix at the center. Also, the center owner has decided on α_i, β_i, $\bar{S}_i$. We assume that these contract terms are set in a competitive market. Tenants choose when (and whether at all) to exercise the option to leave the center.

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[3]These recoveries often have significant value to the center owner, which means that valuations ignoring these recoveries will be too low.

3. Value of a Pure Fixed Rent Contract

We can compute the present value of a pure fixed rent contract as follows. Let $V_i(Q_i, Q_{-i}, X_i(0), D_i(0))$ be tenant i's profit function (in present value terms). This value function is just the expected (discounted) value of all future profits, i.e.,

$$V_i(Q_i, Q_{-i}, X_i(0), D_i(0)) = E_0[\int_0^\infty e^{-r_i t}(S_i(t) - P_i^a(t)) \cdot Q_i(t))dt] \tag{1}$$

where $Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$ and r_i is a constant risk-adjusted discount rate – constant because risk is assumed to be unchanging during the life of the lease, and risk-adjusted because we assume $X_i(t)$ grows at μ_i rather than at a risk-neutral rate. The infinite time horizon structure in (1) implies that there need not be a definite date when the tenant must cease to operate or elect to default.

The problem is to choose a boundary where tenant i is free to terminate its lease (without further cost) in order to maximize (1). We define the Hamilton-Jacobi-Bellman equation by

$$r_i V(Q_i, Q_{-i}, X_i, D_i) = E[(S_i(t) - P_i^a(t)) \cdot Q_i(t))] + \frac{E[dV]}{dt} \tag{2}$$

That is, the return on investment, $r_i V(Q_i, Q_{-i}, X_i, D_i)$, equals the per period cash flow, $E[(S_i(t) - P_i^a(t)) \cdot Q_i(t))]$, plus the expected rate of capital gain, $\frac{E[dV]}{dt}$. Then using Ito's lemma we can write

$$E[dV] = \{\mu_i X_i V_X + \frac{1}{2}\sigma_i^2 X_i^2 V_{XX} - \theta_i D_i V_D + \sum_{j \neq i} p_{ji}\}dt \tag{3}$$

where V_X is the first derivative of V with respect to X_i, V_{XX} is the second derivative of V with respect to X_i, V_D is the first derivative of V with respect to D_i, and p_{ji} is given by

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$$p_{ji} \approx E\left[V_{Q_j}\frac{dQ_j}{dt}\right] \qquad (4)$$

Here p_{ji} represents the expected change in $V(Q_i, Q_{-i}, X_i, D_i)$ due to the possible exit of tenant j from the mall (which could occur when tenant j's retail sales fall below some critical level). p_{ji} has two components: the expected loss if tenant j defaults, V_{Q_j}, and the probability that default will occur, $\frac{dQ_j}{dt}$.[4]

The general solution to (3) is

$$V(Q_i, Y_i) = A_1(Q_i)Y_i^{\theta_i} + B_1(Q_i)Y_i + C_1(Q_i) \qquad (5)$$

where $Y_i = D_i X_i$. The default boundary condition where tenant i is free to exit the mall occurs when Y_i^* falls below a certain multiple k_i of Q_i, where $k_i Q_i$ is an exogenous value, not controlled by uncertainty. The value of $k_i Q_i$ can be viewed as a certain recovery or liquidation value – i.e., what the tenant walks away with when he or she leaves the center. One would expect the value of $k_i Q_i$ to be quite small, since most retailers typically maintain restricted cash accounts and have few tangible assets. Further, because the focus of the paper is on what happens when tenant i finds itself in a situation where one of the major tenants at the center has already gone bankrupt or ceases to operate, or on what happens when the center is losing market share and sales to other centers, no reletting of Q_i is permitted.

The value matching condition at the trigger value of Y_i^* is

$$V(Q_i, Y_i^*) = A_1(Q_i)Y_i^{*\theta_i} + B_1(Q_i)Y_i^* + C_1(Q_i) = k_i Q_i \qquad (6)$$

[4]Reasonable parameter values are: $V_{Q_j} = 0.25$ (from Gatzlaff, Sirmans and Diskin (1994), who conclude that non-anchor tenant rents decline by an estimated 25% after the loss of a major tenant) and $\frac{dQ_j}{dt} = 0.03$ to 0.04 (from our simulation analysis). This suggests a value of p_{ji} in the 0.0075 to 0.0100 range.

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This condition is easy to interpret. It equates the value of the default option to the certain recovery value.

The smooth-pasting condition is

$$\frac{\partial F(Q_i, Y_i^*)}{\partial Y_i^*} = \theta_i A_1(Q_i) Y_i^{*\theta_i - 1} + B_1(Q_i) = 0 \tag{7}$$

where

$$\theta_i = \frac{-\left(\mu_i - \theta_i - \frac{1}{2}\sigma_i^2\right) - \sqrt{\left(\mu_i - \theta_i - \frac{1}{2}\sigma_i^2\right)^2 + 2\sigma_i^2 (r_i + p_i)}}{\sigma_i^2}$$

and

$$p_i = \sum_{j \neq i} p_{ji} \quad \text{and} \quad p_i \geq 0.$$

This condition ensures that the function relating the default option to the certain recovery value at the point of default has zero slope.

The value function that satisfies these two conditions is

$$V(Q_i, Y_i) = \frac{1}{1 - \theta_i} \left(\frac{\alpha_i}{r_i + p_i} + k_i\right) Q_i \left(\frac{Y_i}{Y_i^*}\right)^{\theta_i}$$

$$+ \frac{Q_i^{\lambda_i + 1} \prod_{j \neq i} Q_j^{\gamma_{ji}}}{r_i + \theta_i - \mu_i + p_i} Y_i - \frac{\alpha_i Q_i}{r_i + p_i} \tag{8}$$

where

$$Y_i^* = \frac{\theta_i}{\theta_i - 1} \left(\frac{\alpha_i}{r_i + p_i} + k_i\right) \frac{r_i + \theta_i - \mu_i + p_i}{Q_i^{\lambda_i} \prod_{j \neq i} Q_j^{\gamma_{ji}}}.$$

Equation (8) has an intuitive interpretation. The first term on the RHS of (8) denotes the value of tenant i's default option. The default option is exercised when Y_i hits the critical value Y_i^*. At this point, tenant i is free to leave the center without further cost. The critical trigger value Y_i^* depends on the parameters α_i, k_i, and the optimal tenant mix, Q_{-i}. As we vary α_i and k_i, the critical trigger value Y_i^* varies in the same direction (as does the default option). However, as we vary Q_{-i} (which would alter the externalities among the tenants), the critical trigger value Y_i^* varies in the opposite direction (as does the default option). The default option is discounted at $r_i + p_i$.

The other two terms on the RHS of (8) represent the expected discounted value of sales and the rental flow, respectively. These two terms are discounted at different rates, the first term is discounted at $r_i + \theta_i - \mu_i + p_i$ (since sales are assumed to grow (net of depreciation) geometrically at $\mu_i - \theta_i$), and the second term is discounted at $r_i + p_i$ (since $P_i^a(t) = \alpha_i$ is fixed for all sales).

The novelty in this formulation is the risk premium term, p_i. When tenant j confers no externality on tenant i, and vice versa, the term p_i disappears (i.e., $V_{Q_j} = 0$ in equation (4)), and the discount rate for the cash flows simply becomes $r_i + \theta_i$, which is identical to the well-known Jorgensonian user cost of capital used by Grenadier (1995, 1996) and others to value retail leases. The opposite conclusion holds when tenant i and j confer a positive externality on each other (i.e., when $V_{Q_j} > 0$), assuming, of course, $\frac{dQ_j}{dt} > 0$. In this case, the term p_i is needed to compensate the center owner/tenant for the risk that the sales externality effects at the center (however large) might become negative. We will come back to this point later in detail.

4. Value of a Profit Share Rent Contract

The problem is exactly as before: to maximize

$$V_i(Q_i, Q_{-i}, X_i(0), D_i(0)) = E_0[\int_0^\infty e^{-r_i t}(S_i(t) - P_i^b(t)) \cdot Q_i(t))dt] \qquad (9)$$

tenant i must choose when (and whether at all) to exercise its option to exit the center. The only difference in this model than in the one above is that the tenant pays a base rent α_i plus a profit share payment equal to equal to a certain fraction, β_i, of sales, $\bar{S}_i$.

The point at which the value function $V(Q_i, Q_{-i}, X_i, D_i)$ is maximized now occurs at

$$
\begin{aligned}
V(Q_i, Y_i) = &\frac{1}{1-\theta_i}\left(\frac{\alpha_i - \beta_i \bar{S}_i}{r_i - p_i} + k_i\right)Q_i\left(\frac{Y_i}{Y_i^{**}}\right)^{\theta_i} \\
&+ \frac{(1-\beta_i)Q_i^{\lambda_i+1}\prod_{j\neq i}Q_j^{\gamma_{ji}}}{r_i + \theta_i - \mu_i - p_i}Y_i - \frac{(\alpha_i - \beta_i \bar{S}_i)Q_i}{r_i - p_i}
\end{aligned}
\qquad (10)
$$

where

$$Y_i^{**} = \frac{\theta_i}{\theta_i - 1}\left(\frac{\alpha_i - \beta_i \bar{S}_i}{r_i + p_i} + k_i\right)\frac{r_i + \theta_i - \mu_i - p_i}{(1-\beta_i)Q_i^{\lambda_i}\prod_{j\neq i}Q_j^{\gamma_{ji}}}$$

As we see, the terms on the RHS of (10) are well-defined and can be interpreted as follows. First, $\frac{1}{1-\theta_i}\left(\frac{\alpha_i - \beta_i \bar{S}_i}{r_i - p_i} + k_i\right)Q_i\left(\frac{Y_i}{Y_i^{**}}\right)^{\theta_i}$ is the value of the tenant's default option. Second, $\frac{(1-\beta_i)Q_i^{\lambda_i+1}\prod_{j\neq i}Q_j^{\gamma_{ji}}}{r_i + \theta_i - \mu_i - p_i}Y_i$ is the expected discounted value of sales. Third, $\frac{(\alpha_i - \beta_i \bar{S}_i)Q_i}{r_i - p_i}$ is the expected discounted value of rents.

We now perform some comparative statics, focusing on Y_i^{**}. Y_i^{**} goes up, as $\bar{S}_i$ decreases, as you might have guessed. Also, in certain cases, Y_i^{**} goes up, as β_i goes up. This means

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that tenants are more likely to walk away when they are only getting a limited upside. Mathematically, these results follow from

$$\frac{\partial Y_i^{**}}{\partial S_i} = \frac{\theta_i}{1-\theta_i} \cdot \frac{\beta_i}{r_i + p_i} \cdot \frac{r_i + \rho_i - \mu_i + p_i}{(1-\beta_i)Q_i^{\lambda_i} \prod_{j \neq i} Q_j^{\gamma_{ji}}} < 0$$

$$\frac{\partial Y_i^{**}}{\partial \beta_i} = \frac{\theta_i}{1-\theta_i} \cdot \frac{\bar{S}_i - \alpha_i - (r_i + p_i)k_i}{(1-\beta_i)(r_i + p_i)} \cdot \frac{r_i + \rho_i - \mu_i + p_i}{(1-\beta_i)Q_i^{\lambda_i} \prod_{j \neq i} Q_j^{\gamma_{ji}}} \gtrless 0$$

This latter expression is positive if $\bar{S}_i \leq \alpha_i + (r_i + p_i)k_i$.

5. Value of a Percentage Rent Contract

Now we turn to an analysis of percentage rent contracts. The model in outline form is as follows. We define $V_i(Q_i, Q_{-i}, X_i(0), D_i(0))$ for a percentage rent contract as

$$V_i(Q_i, Q_{-i}, X_i(0), D_i(0)) = E_0[\int_0^\infty e^{-r_i t}(S_i(t) - P_i^c(t)) \cdot Q_i(t))dt] \qquad (11)$$

We need to determine two value functions; a value function describing the value of the lease when $S_i(t) \leq \bar{S}_i$, which we denote as $V_A(Q_i, Y_i)$, and a value describing the value of the lease when $S_i \geq \bar{S}_i(t)$, which we denote as $V_B(Q_i, Y_i)$.

These two value functions are given by

$$V_A(Q_i, Y_i) = A_1(Q_i)Y_i^{\omega_{i1}} + A_2(Q_i)Y_i^{\omega_{i2}} + \frac{Q_i^{\lambda_i+1} \prod_{j \neq i} Q_j^{\gamma_{ji}}}{r_i + \theta_i - \mu_i + p_i}Y_i - \frac{\alpha_i Q_i}{r_i + p_i} \qquad (12)$$

and

$$V_B(Q_i, Y_i) = A_3(Q_i)Y^{\omega_{i1}} + \frac{(1-\beta_i)Q_i^{\lambda_i+1}\prod\limits_{j\neq i}Q_j^{\gamma_{ji}}}{r_i + \theta_i - \mu_i + p_i}Y_i$$
$$- \frac{(\alpha_i - \beta_i\bar{S}_i)Q_i}{r_i + p_i} \tag{13}$$

where

$$\omega_{i1} = \frac{-(\mu_i - \theta_i - \frac{1}{2}\sigma_i^2) - \sqrt{(\mu_i - \theta_i - \frac{1}{2}\sigma_i^2)^2 + 2\sigma_i^2(r_i + p_i)}}{\sigma_i^2} < 0$$

$$\omega_{i2} = \frac{-(\mu_i - \theta_i - \frac{1}{2}\sigma_i^2) - \sqrt{(\mu_i - \theta_i - \frac{1}{2}\sigma_i^2)^2 + 2\sigma_i^2(r_i + p_i)}}{\sigma_i^2} > 1$$

As part of the smooth-pasting condition we will need the expression for $\frac{\partial V_A(Q_i, Y_i^{***})}{\partial Y_i^{***}}$ and $\frac{\partial V_B(Q_i, \bar{Y}_i)}{\partial \bar{Y}_i}$, where Y_i^{***} and $\bar{Y}_i$ are critical points of default. Both value functions must also satisfy the Hamilton-Jacobi-Bellman equation.

At the sales threshold level, $\bar{S}_i$, the value matching condition requires $V_A(Q_i, Y_i) = V_B(Q_i, Y_i)$. To ensure continuity when sales cross the threshold level the smooth-pasting condition requires $\frac{\partial V(Q_i, Y_i^{***})}{\partial Y_i^{***}} = \frac{\partial V(Q_i, \bar{Y}_i)}{\partial \bar{Y}_i}$. Knowing $\bar{S}_i$, then, you can determine $V(Q_i, Y_i)$ by assuming $V(Q_i, Y_i)$ takes one form ($V_A(Q_i, Y_i)$) when $S_i(t) \leq \bar{S}_i$, and takes another form ($V_B(Q_i, Y_i)$) when $S_i(t) \geq \bar{S}_i$.

This insight has important implications. When $S_i(t) \leq \bar{S}_i$ there are four components to value: $A_1(Q_i)Y_i^{\omega_{i1}}$, which is the value of the tenant's default option; $A_2(Q_i)Y_i^{\omega_{i2}}$, which is the center owner's claim on the tenant's sales; $\frac{Q_i^{\lambda_i+1}\prod\limits_{j\neq i}Q_j^{\gamma_{ji}}}{r_i + \theta_i - \mu_i + p_i}Y_i$, which is the discounted present value of sales (discounted at $r_i + \theta_i - \mu_i + p_i$); and $\frac{\alpha_i Q_i}{r_i + p_i}$, which is the discounted present value of base rents (discounted at $r_i + p_i$).

Here $A_1(Q_i)Y_i^{\omega_{i1}}$ can be numerically determined. This term should increase as ω_{i1} approaches Y_i^{***}. There is an explicit solution for the second term, $A_2(Q_i)Y_i^{\omega_{i2}}$, namely,

$$A_2(Q_i) = \frac{\omega_{i1} - 1}{\omega_{i2} - \omega_{i1}} \left[\frac{1}{r_i + p_i} - \frac{1}{r_i + \theta_i - \mu_i + p_i} \right] \beta_i Q_i \bar{S}_i \bar{Y}_i^{-\omega_{i2}}$$

$A_2(Q_i)$ becomes more negative, as $\bar{S}_i$ decreases or $\bar{Y}_i$ increases.

Now ask what happens when $S_i(t) \geq \bar{S}_i$. In this case there are three components to value: $A_3(Q_i)Y^{\omega_{i1}}$, which is the value of tenant's default option; $\dfrac{(1 - \beta_i)Q_i^{\lambda_i+1} \prod\limits_{j \neq i} Q_j^{\gamma_{ji}}}{r_i + \theta_i - \mu_i + p_i} Y_i$, which is the discounted present value of sales; and $\dfrac{(\alpha_i - \beta_i \bar{S}_i)Q_i}{r_i + p_i}$, which is the discounted present value of rents (accounting for the fact that the overage option is in the money).

This formal model makes clear that lease value and the optimal default trigger value depend on a variety of parameters. Furthermore, the model makes clear that lease value and the optimal default trigger value will move in opposite directions: when Y_i^{**} increases, $V(Q_i, Y_i)$ decreases. The comparative statics of Y_i^{**} suggest that Y_i^{**} increases as μ_i, α_i and k_i increase, and that Y_i^{**} decreases as σ_i. We also know that as r_i increases, the value of the lease decreases, causing default to become more likely. These insights serve as a basis for understanding the empirical simulations to come.

6. Parameterization of the Model

The exogenous variables in the model are the following:

1. tenant mix: Q_i and $Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$

2. production and sales externality parameters: λ_i and γ_{ji} for each tenant

3. demand shock parameters: μ_i and σ_i

4. rate of economic depreciation: θ_i

5. lease contract terms: α_i, β_i, and $\bar{S}_i$

6. initial sales level: $S_i(0)$

7. discount rate and risk premium: r_i and p_i

8. liquidation value: k_i

A study of retail leases might take several forms, including the pricing of leases used in community, neighborhood, regional, superregional, fashion/specialty, power, theme/festival, outlet, and lifestyle centers. This paper studies one of these forms viz. neighborhood centers.

We take a representative neighborhood center from a large institutional real estate owner and investor. The center has 34 tenants. The tenant mix is: 2 anchor tenants and 32 non-anchors (see below). We parameterize our model to match this center's lease terms and sales information. This adds a degree of realism to the analysis.

The parameter μ_i is set equal to 3%; our best guess on the nominal growth in retail sales. θ_i is of no real import because $\mu_i - \theta_i$ is what matters in the model; so a set of μ and θ's can be selected to be consistent with any value of V. We set $\theta_i = 0$.

The discount rate r_i and risk premium p_i are set to match the survey data reported in Korpacz, a quarterly survey of real estate investors concerning office, retail, apartment, and industrial returns. We set $r_i = 8\%$ and $p_i = 1\%$, for an overall (unlevered) required return of 9%. (As an aside, Shilling (2003) notes that there seems to be a significant equity risk

premium puzzle in real estate. Here the inclusion of the term p_i helps to explain why this premium might exist, at least for retail properties, that is.) We set the valuation multiple $k_i = 2$. This valuation multiple is consistent with those derived from market transactions as reported by Kozloff (2005).

The remaining parameter values – Q_i and $Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$, λ_i and γ_{ji}, α, β_i, and $\bar{S}$, and $S_i(0)$ – are discussed in more detail in the next two sections.

7. Estimating the Stochastic Sales Externalities among Retail Tenants

7.1 Estimation Model

This section deals with the estimation of the stochastic sales externalities (ρ_{ji}) between tenants. Assume that retailers fall into two categories, anchor and nonanchor tenants. A linear time trend specification for identifying the stochastic component of sales (per square foot) for retailers that are anchor tenants is

$$ln(S_j(t)/Q_j) = a_0 f_1 + a_1 f_2 + a_2 t + a_3 W + \epsilon_j^a(t) \quad j = 1, 2 \tag{14}$$

and that for nonanchor tenants is

$$ln(S_i(t)/Q_i) = b_0 x_1 + b_1 x_2 + b_2 x_3 + b_4 t + b_5 ln(Q_i) + b_6 W + \epsilon_i^b(t) \quad i = 1, \ldots, 32 \tag{15}$$

where f_1 and f_2 are fixed effects dummies (that take the value 1 for anchor tenant j and 0 otherwise), x_1, x_2, and x_3 are tenant size dummy variables equal to 1 when Q_i for tenant i is below 2000 square feet, between 2000 and 4000 square feet, and above 4000 square feet, respectively, t is a time trend (ranging from 1 in September 2003 to 25 in September 2005), W

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is a seasonal dummy variable, which takes a value of 1 in December (since sales in December are significantly higher than other months) and 0 otherwise, and $\epsilon_j^a(t)$ for $j = 1, 2$ and $\epsilon_i^b(t)$ for $i = 1, 2, \ldots, 32$ are i.i.d. error terms, where $\epsilon_j^a(t) \sim N(0, \sigma^2(\epsilon_j^a(t)))$ and $\epsilon_i^b(t) \sim N(0, \sigma^2(\epsilon_i^b(t)))$, respectively.

Our first test is to test if equation (14) applies to both anchor tenants $j = 1, 2$, or whether two different models should be used. For this test, we will construct a Chow test. We first estimate (14) separately, once using only the data for anchor tenant $j = 1$ and the other for using the data for anchor tenant $j = 2$. We then pool our data into one set and estimate equation (14) for both $j = 1, 2$. A Chow test fails to indicate any difference in the two equations. The computed F-value is only 1.12, falling below the level of significance for rejection (p-value $= 0.30$).

Our second test is to test if equation (15) applies to all 32 nonanchor tenants, or whether separate models should be used. Here we run a Chow test with 32 equations. The estimated F-value is 1.18, which fails to reject the null hypothesis that the slope and intercept parameters are distinct (p-value $= 0.31$).

We use equation (14) to estimate the residuals $\epsilon_1^a(t)$ and $\epsilon_2^a(t)$ for anchor tenants $j = 1, 2$ for each period t. We use equation (15) to estimate an average residual $\epsilon^b(t)$ across all nonanchor tenants $i = 1, 2, \ldots, 32$ for each period t. We then regress $\epsilon^b(t)$ on $\epsilon_1^a(t)$ and $\epsilon_2^a(t)$. That is, we estimate

$$\epsilon^b(t) = \tau_0 \epsilon_1^a(t) + \tau_1 \epsilon_2^a(t) + \zeta(t) \tag{16}$$

where $\tau_0 = \sqrt{\rho_1} \dfrac{\sigma(\epsilon^b(t))}{\sigma(\epsilon_1^a(t))}$, $\tau_1 = \sqrt{\rho_2} \dfrac{\sigma(\epsilon^b(t))}{\sigma(\epsilon_2^a(t))}$, $\zeta(t) = \sqrt{1 - \rho_1 - \rho_2} \dfrac{\sigma(\epsilon^b(t))}{\sigma(\zeta(t))} \psi(t)$, $\rho_1 = Corr(\epsilon^b(t),$

$\epsilon_1^a(t))$, $\rho_2 = Corr(\epsilon^b(t), \epsilon_2^a(t))$, and $\psi(t)$ is an error term. This equation follows from the definition of $Corr(\epsilon^b(t), \epsilon_i^a(t))$ for $i = 1, 2$. From (14) and (15), we can obtain estimates for $\sigma(\epsilon_1^a(t))$, $\sigma(\epsilon_2^a(t))$, and $\sigma(\epsilon^b(t))$. From (16), we can obtain an estimate of $\sigma(\psi(t))$. This allows us to solve for ρ_1 and ρ_2 (the extent to which higher anchor tenant sales will result in higher non-anchor tenant sales).

An immediate consequence of this result is that we can use ρ_1 and ρ_2 as two alternative parameter values of the instantaneous correlation coefficient ρ_{ji}, defined by

$$dZ_j dZ_i = \rho_{ji} dt \tag{17}$$

where dZ_j and dZ_i are Wiener processes in the sales functions for tenants j (anchor tenant) and i (nonanchor tenants), respectively. ρ_{ji} is a key parameter in our analysis. We generally expect ρ_{ji} to be positive and significant in most shopping centers (which is why the loss of a major tenant often has a big impact on a shopping center). Below, we report on our estimates of ρ_1 and ρ_2.

7.2 Empirical Illustration

The data used to estimate (14) and (15) are summarized in table 1. Our data include information on Q_i and $Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$, $S_i(t)$, and α, β_i and $\bar{S}$; although summary information on only the first four of these variables is reported in table 1. We also have data on the estimated market value of the center, along with property capitalization rate information.

Using this data, we have estimated the linear time trend models as specified in (14) and (15): for the two largest tenants in the sample (which we take to be the anchor tenants), and

Category by size	No. of retail-ers	Average size in sf	Size std dev	Average sales psf	Sales std dev	Average rent psf	Rent std dev
Below 2000 sf	8	1379	236	363	228	31.23	5.89
2000-4000 sf	13	2977	693	337	219	25.94	1.73
4000-7000 sf	11	5264	804	322	184	23.92	2.36
Above 7000 sf	2	8227	221	187	81	22.13	2.02
Total	34						

Table 1: Characteristics of the sample. All data are annualized. The sample extends from September 2003 to September 2005. Tenants are grouped according to size. The center meets the classification requirement for neighborhood centers by having about 125,000 square feet. The two largest tenants are taken to be the anchor tenants. The remaining tenants are assumed to be nonanchor tenants. Tests are conducted to determine whether inside these two groups the tenants are different, and whether across these two groups the tenants are different.

for all other tenants (which are taken to be nonanchor tenants). There is not much point of grouping the data in other ways, given that we are using a small sample. Plus, we don't mean to misrepresent ourselves here. The basic objective is simply to come up with a "ball park" estimate of the stochastic sales externalities among retail tenants in a shopping center. We can always experiment a little with higher or lower values of this parameter.

In table 2, we show the results of estimating (14) and (15). The central parameters of interest are the root MSEs reported in column 1 and 2 (which are the standard deviations of the error terms $\sigma(\epsilon_i^a(t))$ and $\sigma(\epsilon^b(t))$ in (14) and (15)). The MSE for anchor tenants is 0.25. Compared to a MSE for nonanchor tenants of 0.64. To put these numbers in perspective, we have also examined data from Eppli et al. (2001) involving 3,445 regional and superregional shopping center leases with percentage rent clauses. The data are for 1995. The average volatility of sales for anchor tenants was 0.20 and 0.26 for nonanchor tenants. This difference can be explained in part as follows. The Eppli et al. sales data are annual. The sales data

	Anchor	Nonanchor
f_1	2.76	
	(31.62)	
f_2	2.33	
	(26.53)	
x_1		4.51
		(5.24)
x_2		4.79
		(5.09)
x_3		4.75
		(4.66)
t	0.0045	0.0061
	(0.84)	(1.71)
$ln(Q_i)$		-0.21
		(-1.76)
W	0.65	0.63
	(4.90)	(6.87)
Root MSE	0.25	0.64
R^2	0.99	0.96

Table 2: OLS estimates. Dependent variable is $ln(S_i(t)/Q_i)$. $S_i(t) =$ retail sales for tenant i. $Q_i =$ size of tenant i in square feet. f_1 and f_2 are fixed effects dummies for anchor tenant 1 and 2. x_1, x_2, and x_3 are tenant size dummy variables equal to 1 when Q_i for tenant i is below 2,000 square feet, between 2,000 and 4,000 square feet, and above 4,000 square feet, respectively. t is a time trend variable. W is a seasonal dummy variable, which takes a value of 1 in December and 0 otherwise. t-statistics for estimated parameters are presented in parameters. Root $MSE =$ root mean squared error. $R^2 =$ the ratio of the explained variation to the total variation.

used here are monthly, and monthly data are more volatile than annual averages.

The time trend coefficients in table 2 vary between 0.0045 and 0.0061. This suggests an annualized growth rate in sales of between 5.3 and 7.4%. However, this effect seems weak (especially for anchor tenants). This may be attributable to the macroeconomic environment during the period we investigate.

We now use these residuals to estimate equation (16). The results are reported in table 3.

	Coefficient estimates
$\epsilon_1^a(t)$	0.26
	(2.19)
$\epsilon_2^a(t)$	0.23
	(1.98)
Root MSE	0.64
R^2	0.02

Table 3: Estimate of the Stochastic Sales Externality among Retail Tenants. Dependent variable is $\epsilon^b(t)$. $\epsilon^b(t)$ = residual from equation (16). $\epsilon_j^a(t)$ for j = 1, 2 = residual from equation (14). t-statistics for estimated parameters are presented in parameters. Root MSE = root mean squared error. R^2 = the ratio of the explained variation to the total variation.

The R^2 for this regression is 0.02, which is quite low, but not unexpectedly so (given that we are regressing a residual on residuals). The coefficients on $\epsilon_1^a(t)$ and $\epsilon_2^a(t)$ have the expected sign and are significant. For values of $\sigma(\epsilon_1^a(t)) = 0.35$, $\sigma(\epsilon_2^a(t)) = 0.34$, $\sigma(\epsilon^b(t)) = 0.64$, and $\sigma(\zeta(t)) = 0.64$, the coefficients on $\epsilon_1^a(t)$ and $\epsilon_2^a(t)$ imply $\rho_1 = 0.10$ and $\rho_2 = 0.09$. These results are rather dramatic and reveal that a 10% increase in anchor tenant sales per square foot a year should increase nonanchor tenant sales per square foot by about 1.0% a year. Thus, if the anchor tenants were doing \$200 per square foot worth of retail volume a year, and occupying 20,000 (total) square feet of space, an increase of 10% in anchor tenant sales per square foot would increase the anchor tenants' total business by \$400,000 a year. Similarly, if the nonanchor tenants were doing \$300 per square foot worth of retail volume a year, and occupying 120,000 (total) square feet of space, an increase of 1% in nonanchor tenant sales per square foot would increase the nonanchor tenants' total business by \$360,000 a year. Thus, together total business at the center would increase by \$400,000 + \$360,000 = \$760,000 a

year, which is an additional \$0.90 a year in purchases at the nonanchor tenant stores per every additional \$1 a year in purchases at the anchor tenant stores.

We can now use this information about ρ_1 and ρ_2 in a ceteris paribus analysis to simulate the distribution of sales that would prevail if anchor tenants convey a positive sales externality on nonanchor tenants. The simulated sales are generated assuming $\mu = .03$, $\sigma = .30$, $\rho_{ij} = 0$ (i.e., nonanchor tenants confer no externalities on the anchors), and $S(0) = \$200$ per square foot for a typical anchor tenant and $\mu = .03$, $\sigma = .50$, $\rho_{ji} = 0.20$, and $S(0) = \$300$ per square foot for a nonanchor tenant.
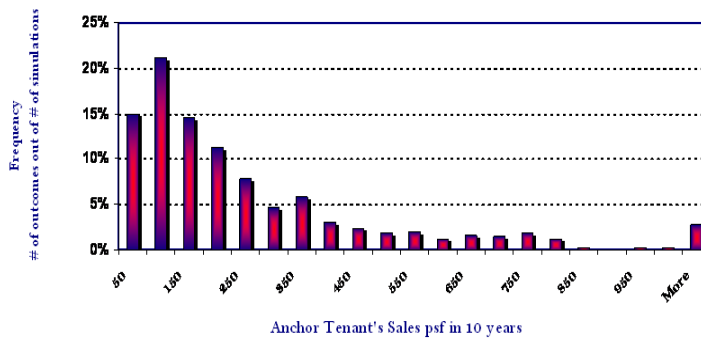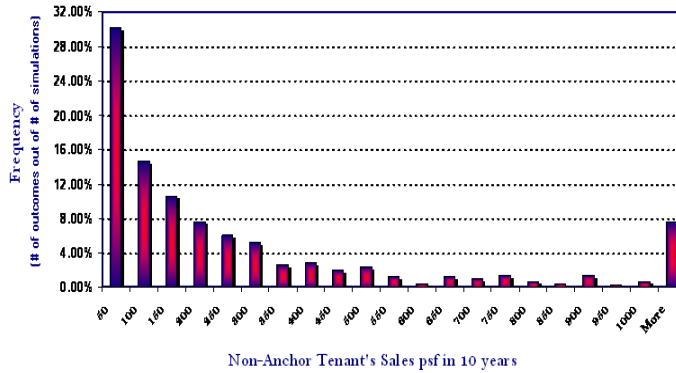

Figure 1: Simulated anchor tenant sales at end of year 10. Results obtained from a Monte Carlo simulation forecasted out to year 10. Vertical axis: Number of outcomes out of number of simulations. Horizontal axis: Sales per square.

Figure 1 plots the sales that could occur for an anchor tenant; these sales are obtained from a Monte Carlo simulation forecasted out to year 10. The distribution is lognormal. Expected sales in year 10 are about \$250; the 99% exposure ranges to about \$850 per square

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foot.

Figure 2: Simulated nonanchor tenant sales at end of year 10. Results obtained from a Monte Carlo simulation forecasted out to year 10. Vertical axis: Number of outcomes out of number of simulations. Horizontal axis: Sales per square.

Figure 2 depicts the sales for a nonanchor tenant. As expected, the distribution has fatter tails than a lognormal distribution, but without taller peaks. The expected sales in year 10 is about $350 per square foot. The 99% exposure ranges to more than $1000 per square foot, and the measures in both tails of the distribution are over two times the expected exposure for an anchor tenant. The fat tails are associated with dependence on ρ_{ji}. We find that the consequences of ignoring this externality effect are enormous. The reason is simply that as ρ_{ji} approaches zero, the distribution of $S_i(t)$ for nonanchor tenants approaches a lognormal distribution (and this will seriously bias any estimates made of the risk premium p_i).

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8. Specification of Production Parameters and Lease Contract Terms

There are eight remaining exogenous variables that need to be specified: the four production parameters Q_i, $Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$, λ_i, and γ_{ji}, and the four lease contract terms α_i, β_i, $\bar{S}$, and $S_i(0)$. The values of λ_i and γ_{ji} are not observed; however, these values (along with Q_i, $Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$) can be inferred. Because $S_i(t) = D_i(t) \cdot X_i(t) Q_i^{\lambda_i} \prod_{j \neq i} Q_j^{\gamma_{ji}}$ applies for all t, we can substitute the value $S_i(0)$ for $Y_i Q_i^{\lambda_i} \prod_{j \neq i} Q_j^{\gamma_{ji}}$, and plug this result back into our model; thereby eliminating the specification of Q_i, $Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$, λ_i, and γ_{ji}.

The four lease contract terms α_i, β_i, $\bar{S}$, and $S_i(0)$ are chosen to be consistent with the data. We set $\alpha_i = \$30$ per square foot, $\beta_i = 0.07$, $\bar{S} = \$400$ per square foot, and $S_i(0) = \$300$ per square foot for percentage rent contracts. The simulation model is then coded with the demand shock parameters, the rate of economic depreciation, the initial sales level, the discount rate and risk premium, the liquidation value, and an assumed value of $V = \$265$ per square foot and solved for an equivalent fixed rent for a pure fixed rent contract. The solution was $\$40.26$ per square assuming a nonexistent probability of default.

9. Some Simulation Results

Here we present a number of simulations to illustrate the basic workings of the model.

9.1 Increase in Sales Percentage Rate

Figure 3 depicts the value of four different lease contracts, and illustrates the effect of varying the percentage rental rate for each one of these leases. The four different lease contracts depicted are: $op(1,0)$ denotes a pure fixed rent contract with default; $op(0,0)$ denotes a pure fixed rent contract without default; we use $op(1,1)$ to denote a percentage rent contract

with default; and $op(0,1)$ to denote a percentage rent contract without default. This vector

notation is convenient (and will be used throughout the remainder of the paper), the first

argument denotes with default $(= 1)$ or without default $(= 0)$; the second argument denotes

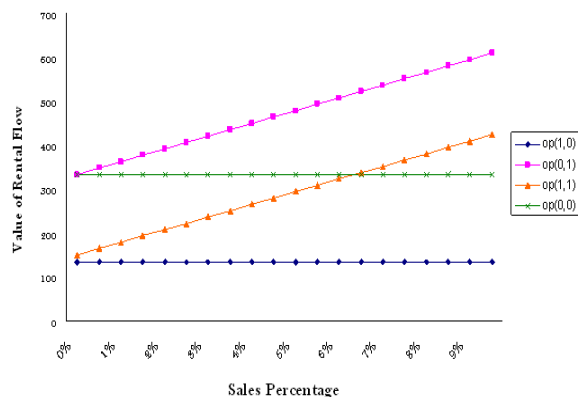a pure fixed rent contract $(= 0)$ or a percentage rent contract $(= 1)$.

Figure 3: Simulated increase in sales percentage rate on the value of a retail lease. Results obtained from a Monte Carlo simulation. Vertical axis: Value of retail lease (V). Horizontal axis: Sales percentage rate (β_i). $op(1,0)$ denotes a pure fixed rent contract with default; $op(0,0)$ denotes a pure fixed rent contract without default; we use $op(1,1)$ to denote a percentage rent contract with default; and $op(0,1)$ to denote a percentage rent contract without default.

Each point on the graph should be thought of as a separate lease contract with a different

sales percentage rate. The value of a pure (default-free) fixed rent contract in this case is

invariant with respect to a change in sales percentage rate β_i; which is no big surprise. Also

no big surprise, $op(0,0$ shows up in $\beta - V$ space as above $op(1,0)$. This comparison is a pretty

trivial example.

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Comparing $op(0,1)$ and $op(0,0)$ is, however, nontrivial. The results suggest (ignoring default) that a percentage rent clause is of positive value to a shopping center owner, but negative to a tenant. Furthermore, the results suggest that a percentage rent clause can perhaps add substantial value; maybe as much as 50% or more.

This comparison, however, overlooks a vital fact – the tenant's embedded default option. This embedded option to leave the center is of positive value to the tenant, but negative to the shopping center owner. Furthermore, for small values of β_i, the value of this option can more than offset the full value of percentage rent clause, causing $op(1,1)$ to fall below $op(0,0)$. For large values of β_i, the opposite result holds: the value of the percentage rent clause more than offsets the tenant's option to default; which makes sense. The two options are not linearly related; together, they are a compound option.

Next, if you think about it, what figure 3 shows is that for moderate values of β_i the values of these two options are such that one may simply give them little or no weight for purposes of, say, financing, and simply discount the fixed guaranteed minimum rent to arrive at value (compare $op(1,1)$ and $op(0,0)$ for moderate values of β_i). This finding, oddly enough, helps to explain why, in practice, many lenders give little or no value to straight percentage of sales rentals for the purpose of financing. Oddly, enough, these lenders appear to be getting values right, but perhaps for the wrong reasons.

What is also interesting about figure 3 is that it offers a somewhat latent role for percentage of sales rentals: namely, to offset the negative value to center owners of the tenant's option to default.

9.2 Increase in Volatility

Figure 4 depicts the value of a shopping center V as a function of volatility σ_i. Shopping

center value V for a pure fixed rent contract assuming a nonexistent probability of default is

unaffected by a change in volatility σ_i. Here we imagine a world, where tenants are subject

to resource balance and participation constraints. Also, if tenants were to default on a retail

lease, we imagine a world where they make a large buyout payment to the center owner.

Hence, depending on these constraints, this will determine the relative positioning of $op(0,0)$

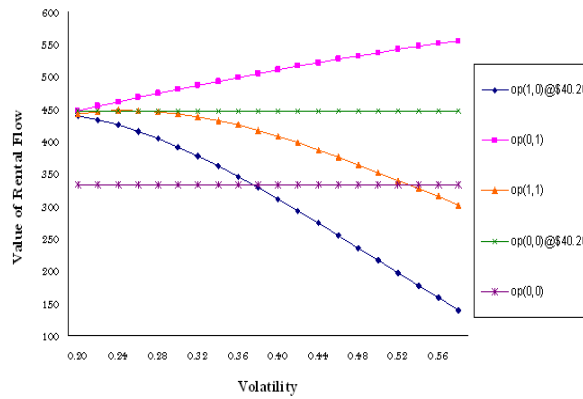and $op(1,0)$ on a $\sigma - V$ graph, but not the slopes of $op(0,0)$ and $op(1,0)$.

Figure 4: Simulated increase in volatility on the value of a retail lease. Results obtained from a Monte Carlo simulation. Vertical axis: Value of retail lease (V). Horizontal axis: Sales volatility (σ_i). $op(1,0)$ denotes a pure fixed rent contract with default; $op(0,0)$ denotes a pure fixed rent contract without default; we use $op(1,1)$ to denote a percentage rent contract with default; and $op(0,1)$ to denote a percentage rent contract without default.

V for a percentage rent contract, however, declines as σ increases. Others suggest that V should increase as σ increases; uncertainty raises the value of a percentage rent clause. This prediction ignores, however, risk considerations and possible externalities. After controlling for these two effects, we find that V decreases as σ_i increases. This occurs because the tenant's option to default more than offsets the value of a percentage rent clause. This is especially true for moderate to large values of σ_i; compare $op(1,1)$ to $op(1,0)$ for values of σ_i between, say, 0.4 and 0.6. For small values of σ_i, however, V appears to increase as σ_i increases, but by not by much; compare $op(1,1)$ to $op(0,1)$ for values of σ_i between 0.2 and 0.25. This, again, illustrates the point that, together, the tenant's option to default and the percentage rent clause are compound options.

9.3 Increase in Sales Percentage Rate and Sales Threshold Level

Figure 5 gives the results of how shopping center value V varies with sales percentage rate β_i and sales threshold level $\bar{S}_i$, holding all else equal. The results here are interesting. Owing to a change in both β_i and $\bar{S}_i$, we find that shopping center value V decreases as both β_i and $\bar{S}_i$ increase.

We would anticipate that an increase in β_i should raise V by raising the value of the overage option (as shown in figure 3). However, when $\bar{S}_i$ increases as well, or when the overage option is way out-of-the-money, an increase in β_i can affect the tenant's default option more than the overage option, thereby causing property values to fall.

As a result a nontrivial rent increase would be needed to keep V from falling. Hence, in this case base and sales percentage rents may move positively together, and not negatively as others have suggested.
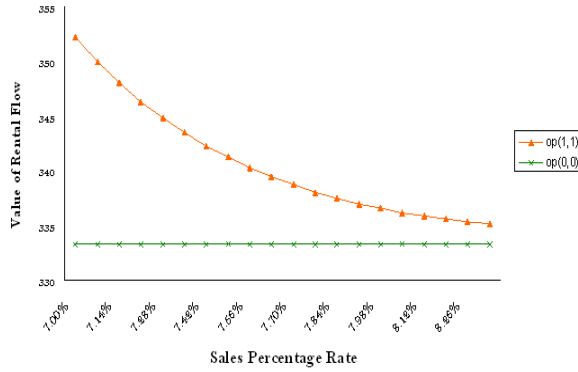
Figure 5: Simulated increase in sales percentage rate and sales threshold level on the value of a retail lease. Results obtained from a Monte Carlo simulation. Vertical axis: Value of retail lease (V). Horizontal axis: Sales percentage rate (β_i). Not shown: Change in sales threshold level. $op(0,0)$ denotes a pure fixed rent contract without default; we use $op(1,1)$ to denote a percentage rent contract with default.

9.4 Increase in Sales Drift Term and Discount Rates

Here we simulate how property capitalization rates vary with discount rates r_i and growth in sales μ_i. The results are illustrated in figure 6. This figure is drawn assuming that as μ_i goes up, r_i goes up, but not by much. All other variables are assumed to be held constant.

The results in figure 6 are easy to explain. Generally, because of a large increase in μ_i, the value of the tenant's option to default falls. This, in turn, leads to a significant increase in the value of the lease. In contrast, as r_i increases, the value of the lease decreases, but only marginally so.
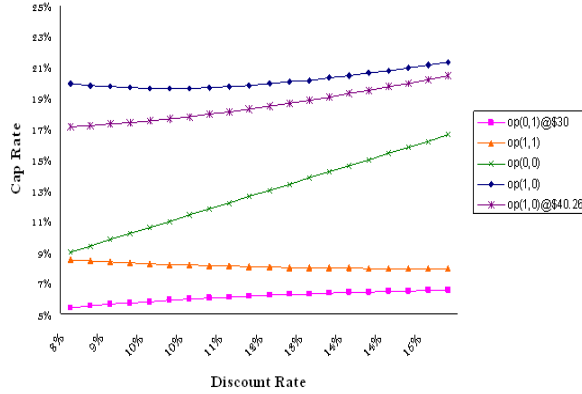
Figure 6: Simulated increase in sales drift term and discount rates on the property capitalization rate. Results obtained from a Monte Carlo simulation. Vertical axis: Property capitalization rate. Horizontal axis: Sales drift term (μ_i). Not shown: Change in discount rate. $op(1,0)$ denotes a pure fixed rent contract with default; $op(0,0)$ denotes a pure fixed rent contract without default; we use $op(1,1)$ to denote a percentage rent contract with default; and $op(0,1)$ to denote a percentage rent contract without default.

Because the two effects are not equal, property values rise, and property capitalization rates decrease. In this way interest rates and property capitalization rates become negatively, as opposed to positively, related to each other.

We next examine how default varies with an increase in r_i. In the case where tenants pay nothing but an equivalent annual base rent of $40.26 per square foot, the optimal trigger value range is between $27 and $32 per square foot (see figure 7). Holding μ_i constant, this optimal trigger value increases as r_i increases. This occurs for the following reason: as r_i increases, it now become more costly to delay the decision to default. This in turn raises

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the optimal trigger value (although the range is quite small for the parameter values used in these calculations). Next, a ceteris paribus increase in the optimal trigger value lowers the property price, which would raise the capitalization rate. So with a large enough increase in r_i, interest rates and property capitalization rates could go back to being positively related to each other.
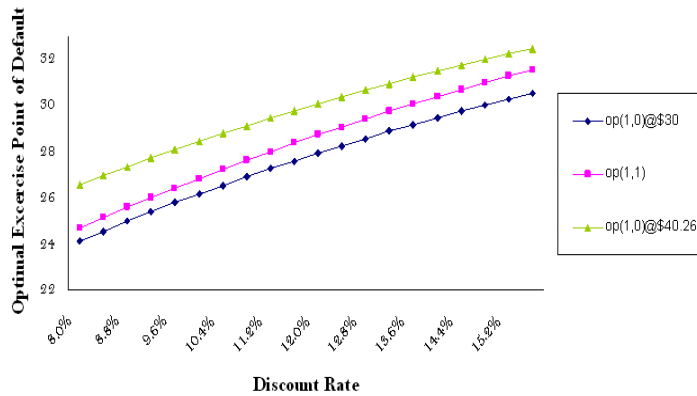

Figure 7: Optimal trigger value as a function of the discount rate. Results obtained from a Monte Carlo simulation. Vertical axis: Discount rate (r). Horizontal axis: Optimal default trigger value. $op(1,0)$ denotes a pure fixed rent contract with default; we use $op(1,1)$ to denote a percentage rent contract with default; and $op(0,1)$ to denote a percentage rent contract without default.

By comparison when tenants pay a percentage rent, the optimal trigger values are generally between \$25 and \$30 per square foot. These lower trigger values mean that property prices are higher (less default risk) and capitalization rates are lower.

Since the optimal trigger values are declining in opportunity costs, property values are

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even higher for pure fixed rent contracts at an annual base rent of $30 per square foot, which also makes sense. But, of course, we don't expect to observe such contracts in the market.

9.5 Increase in Stochastic Sales Externality Risk Premium

At the start of the paper we said that the usual theory for measuring the user cost of a retail lease as the sum of the expected risk-adjusted discount rate, plus the depreciation incurred and minus the rate of asset price change no longer applies. Rather the user cost needs to include a risk premium to compensate the owner/tenant for the risk that the sales externality effects at the center could dissipate or disappear over time. We also said that ignoring this risk premium may significantly overestimate shopping center value. Here we simulate this basic idea in figure 8, where we plot shopping center value V as a function of the risk premium p_i.

This is a useful way to sum up the model. The key point is that V is a downward function of p_i, which means that V may be significantly overstated if p_i is left out of the model. Of course, another way to think about figure 8 is that shopping center owners have a natural incentive to coordinate the range of shops over the life of the shopping center in order to minimize p_i (or, equivalently, to maximize V). Furthermore, this occurs irrespective of a pure fixed or percentage rent contract. The counter argument is that shopping center owners have some influence over tenant revenues because they control both tenant mix and location within the center, and some tenant mixes/positions provide more synergy and positive externalities than others. Hence, tieing the shopping center owner lease payment to the performance of their tenants encourages the owner to maximize total sales within the center. These latter models, however, do not allow for a default option.
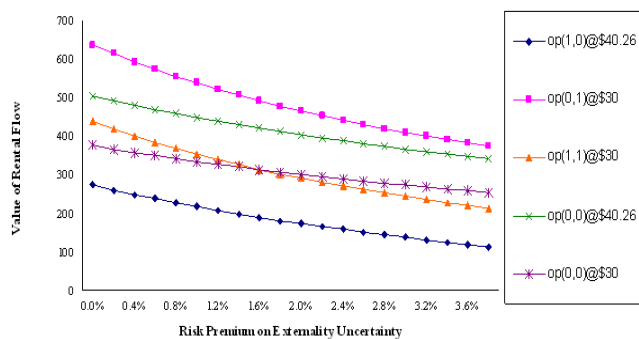
Figure 8: Simulated increase in stochastic sales externality risk premium on the value of a retail lease. Results obtained from a Monte Carlo simulation. Vertical axis: Value of retail lease (V). Horizontal axis: Stochastic sales externality risk premium (p_i). Not shown: Change in discount rate. $op(1,0)$ denotes a pure fixed rent contract with default; $op(0,0)$ denotes a pure fixed rent contract without default; we use $op(1,1)$ to denote a percentage rent contract with default; and $op(0,1)$ to denote a percentage rent contract without default.

10. Conclusions

In this paper we have provided a general formulation for valuing a retail shopping center under uncertainty. We then analyzed the effects of changes in sales percentage rates, uncertainty, sales threshold levels, sales drift terms, discount rates, and risk premia on valuation.

Our model differs from previous studies in a number of ways. Previous studies in the literature are concerned either with inter-store externalities (and with showing that center owners need to allocate as much as 60% of the space at the center to large major retailers at low rents in order to maximize total revenues at the center), or with the pricing of retail

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leases in a framework in which sales follow a stochastic process. These models typically ignore feedback effects, such as the loss of a major tenant (or a decline in the tenant's customer drawing power) on sales, occupancy rates, and rent levels of the other tenants. We in this paper are concerned with an environment in which sales are stochastic; rents are set in a competitive market; tenants confer a positive externality on each other; and in which defaults can occur. We then use the model to show that the usual certainty theory for measuring the user cost of a retail lease as the sum of the expected risk-adjusted discount rate, plus the depreciation incurred and minus the rate of asset price change no longer applies; instead, one needs to discount at a slightly higher discount rate to compensate the owner/tenant for the risk that the sales externality effects at the center could dissipate or disappear over time.

Several other features of the above theory are perhaps worthy of notice. First, ignoring these feedback effects means that shopping center valuations are perhaps too high and property capitalization rates are too low. Second, it is a feature of the theory that a percentage rent clause is of positive value to a shopping center owner, but negative to a tenant. It is also a feature of the theory that the tenant's embedded default option is of positive value to the tenant, but negative to the shopping center owner. Generally, we find that these two options offset one another, thereby offering a perhaps somewhat latent role for percentage of sales rentals: to offset the negative value to center owners of the tenant's embedded default option.

It is also a feature of the theory that shopping center owners have a natural incentive to coordinate the merchandise mix of the shops over the life of the shopping center in order to minimize the value of the tenant's embedded default option. This is an important point, a point that has gone unnoticed.

Even though the model is relatively simple, it helps to explain a variety of phenomena

ranging from an increase in sales volatility may reduce, rather than increase, shopping center value, as others have suggested; why base and percentage rents may move positively together, and not negatively as others have suggested; why retail property capitalization rates and interest rates can be negatively, as opposed to positively, related with each other; and why leases with a percentage of sales rental are often given little or no value for purposes of financing by lenders, regardless of whether or not the tenant is a national chain with substantial net worth.

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Symbol	Definition
Q_i	Tenant size in square feet
$Q_{-i} = \{Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n\}$	Size of other tenants in square feet
$V_i(Q_i, Q_{-i}, X_i(0), D_i(0))$	Tenant i's profit function (in present value terms)
$S_i(t)$	Sales per square foot for tenant i
λ_i and γ_{ji}	Elasticities of production
X_i	Demand shocks
μ_i	Instantaneous conditional expected percentage change in sales per unit time
σ_i	Instantaneous conditional standard deviation in sales per unit time
$dZ_i(t)$	A standard Weiner process
θ_i	Economic rate of depreciation
α_i	Base rent per square foot for tenant i
β_i	Sales percentage rental rate
$\bar{S}_i$	Threshold sales level
r_i	Discount rate
p_i	Risk premium
k_i	Liquidation value
ρ_{ij}	Stochastic sales externality from tenant i to j

Appendix: Symbols Used in the Paper.